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LISA long-arm interferometry: an alternative frequency pre-stabilization system

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Abstract
Laser frequency noise is a significant noise source which couples into the main science measurement of the Laser Interferometer Space Antenna via the mismatch between the interferometer arm lengths. In this paper we discuss the application of an unequal pathlength heterodyne Mach–Zehnder interferometer to measure and actively stablize the master laser frequency as used in LISA Pathfinder. In comparison with an optical cavity or atomic reference the technique has a wide operating range and does not require a complex lock acquisition procedure. Frequency tuning can be provided by purely electronic means and does not require physically changing the pathlength (or resonance frequency) of the frequency reference and can therefore be combined with arm locking in a straightforward manner.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

The Laser Interferometer Space Antenna (LISA) aims to detect gravitational waves in the 100 μHz to 1 Hz frequency band by measuring the distances between freely falling proof masses enclosed within three identical spacecraft in a triangular constellation [1]. The 5 million km inter-spacecraft distances will be measured using laser interferometry with close to shot-noise-limited precision. Whilst the orbits of the spacecraft are designed to keep the separation between the three spacecraft equal and as constant as possible without station-keeping, a slowly changing mismatch on the order of ±60 000 km is unavoidable [2]. This large arm length mismatch couples laser frequency noise with the differential single-pass length measurement between two adjacent arms according to the following expression:

$$\sqrt{S_x(f)} = \left|\mathrm{sinc}\left(\frac{2\pi f \Delta L}{c}\right)\right|^{-1} \frac{\Delta L}{\nu}\sqrt{S_\nu(f)}, \tag{1}$$

where ΔL is the arm length mismatch, ν is the laser frequency, c is the speed of light, $\sqrt{S_x(f)}$ is the displacement amplitude spectral density in m $\mathrm{Hz}^{-1/2}$ and $\sqrt{S_\nu(f)}$ is the frequency amplitude spectral density in Hz $\mathrm{Hz}^{-1/2}$. For $f \ll c(2\pi\Delta L)^{-1}$ equation (1) can be approximated by

$$\sqrt{S_x(f)} \approx \frac{\Delta L}{\nu}\sqrt{S_\nu(f)}. \tag{2}$$

The coupling into the phase measurement is therefore

$$\sqrt{S_\phi(f)} \approx 2\pi\frac{\Delta L}{c}\sqrt{S_\nu(f)}. \tag{3}$$

In the baseline configuration where offset phase locking is used, a single interferometer arm is essentially a Mach–Zehnder interferometer with an enormous mismatch of 10 million km. The coupling of the laser frequency noise to a single arm measurement is on the order of 210 rad Hz^{-1}. With the baseline pre-stabilized frequency noise level of 30 Hz $\mathrm{Hz}^{-1/2}$ [3], the phase noise of the beatnote to be measured is 6300 rad $\mathrm{Hz}^{-1/2}$. This comparatively large phase noise must be measured by the phasemeter with a fidelity of approximately 6 μrad $\mathrm{Hz}^{-1/2}$ [4, 5]. It is only after subsequent active stabilization with arm-locking and processing the raw measurements using a post-processing technique called time-delay interferometry (see e.g. [6, 7]) that the laser frequency noise is removed.

Time-delay interferometry (TDI) 'synthesizes' an equal arm length interferometer by appropriately combining the raw signals with delayed versions of the same signals. To process the raw data using TDI requires an accurate knowledge of the propagation delay in each arm. The main phase measurement observes the picometer level *fluctuations* only in the measurement band and does not directly provide a measurement of the absolute propagation delay. One approach is to implement a dedicated ranging system using pseudo-random codes which are phase modulated onto the main carrier. The absolute separation of the spacecraft can be determined by tracking the phase of these codes at the receiving spacecraft. Preliminary results with a laboratory prototype have shown sub-metre resolution [8].

An alternative method of inferring the propagation delay is to minimize the noise power in the TDI outputs [9]. An extension of this approach is to modulate the laser frequency at the edge of the measurement band and adjust the delays used in TDI to minimize this peak in the final output [10]. Initial estimates of the resolution are on the sub-metre level.

Assuming that a contribution of 4 pm $\mathrm{Hz}^{-1/2}$ is allocated to residual frequency noise, equation (1) may be inverted to compute the allowable laser frequency noise for a given arm length mismatch. Figure 1 shows this for a mismatch of 60 000 km and for TDI combined with 1 m ranging accuracy (which corresponds to a 2 m effective arm length mismatch in the worst case). A free-running laser will not meet the requirement with a 1 m ranging accuracy and additional stabilization is needed.

There are a number of possible ways to stabilize the laser frequency including stabilization to a reference cavity [12] or atomic reference [11], and arm locking [13]. A tunable pre-stabilization system is desirable if arm locking is used. Sideband locking can be used with a fixed reference cavity in order to combine the frequency stability of the reference cavity with tunabilty [14].

In this paper we analyse an alternative laser frequency pre-stabilization system, based on that used in the LISA technology package (LTP) on board LISA Pathfinder [15, 16]. Unlike

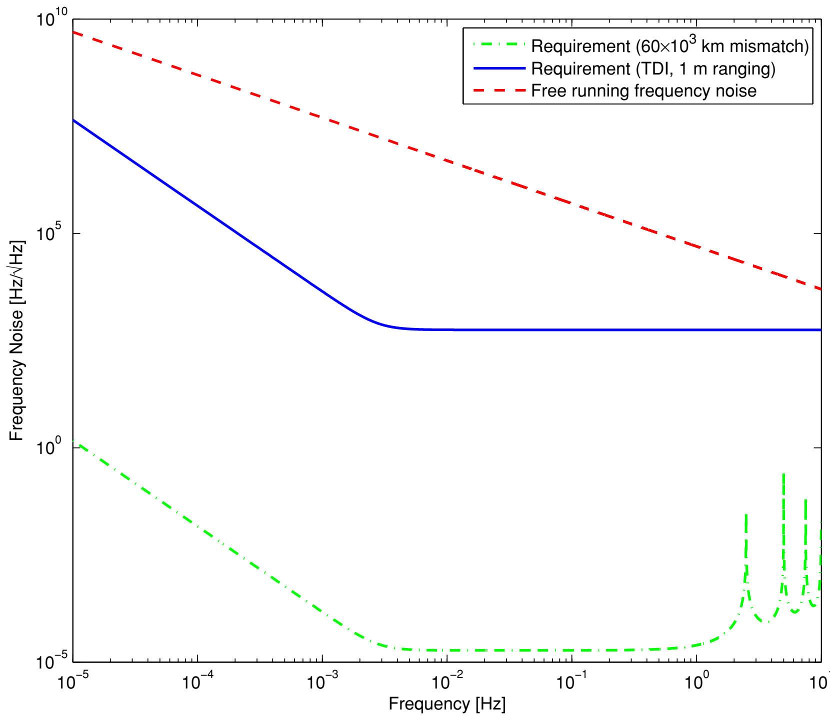

Figure 1. Frequency noise requirements for an arm length mismatch of 60 000 km (20 μHz Hz$^{-1/2}$ at 0.1 Hz) and with TDI combined with 1 m ranging accuracy (560 Hz Hz$^{-1/2}$ at 0.1 Hz). Also shown for comparison is the typical free-running laser frequency for Nd:YAG NPRO lasers.

stabilization to a reference cavity or an atomic reference, no electro-optic modulators, acousto-optic modulators or high power RF electronics are required. The main additional components are a few extra beamsplitters and mirrors on the optical bench and a few extra photodetectors and phasemeter channels. The cleanliness requirements are the same as that of the main optical bench and no additional thermally stable vacuum chamber would be required.

2. 'LTP-style' frequency pre-stabilization

Figure 2 shows a schematic of how an 'LTP-style' pre-stabilization could be implemented on the LISA optical bench. The reference interferometer measures the phase difference (ϕ_R) between the lasers on adjacent optical benches in one LISA satellite. This reference interferometer already exists in the current LISA optical bench design. To measure the frequency noise of the master laser, an additional interferometer with unequal pathlengths, as will be implemented in LISA Pathfinder [15, 16], can be placed on the optical bench. The additional components required include

- single-element photodetectors (2× for redundancy),
- fast (⩾MHz) phasemeter channels (2× for redundancy),
- beamsplitter(s) and mirrors on the optical bench.

The main difference between this system and that of LTP is that the beatnote is generated by interference with an offset phase-locked laser at a variable frequency difference of 2 . . . 20 MHz

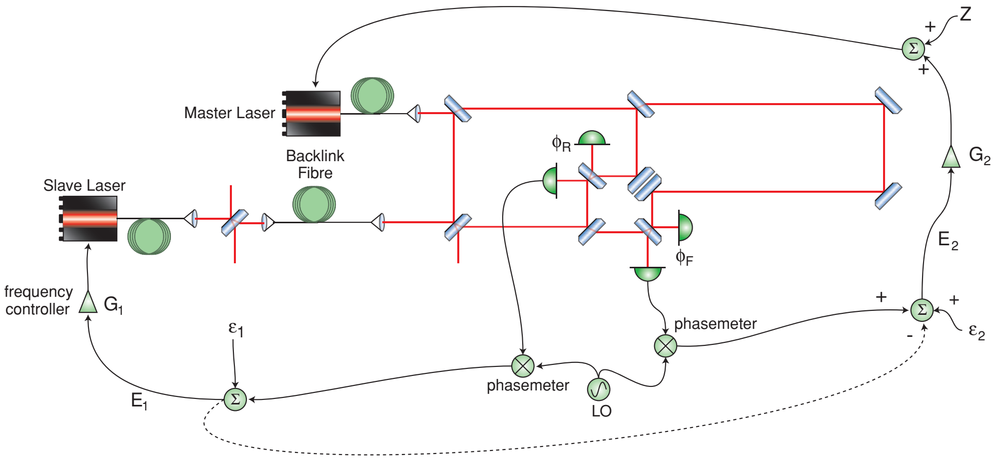

Figure 2. Simplified system layout. The interferometer readout labeled ϕ_R exists in the current LISA optical bench design. The interferometer readout labeled ϕ_F is the readout for the proposed additional interferometer for measuring and actively suppressing the laser frequency noise.

rather than interference between two beams produced by acousto-optic modulators at a constant frequency difference of 1 . . . 2 kHz.

The output of this additional interferometer (ϕ_F) can not only be used to measure the frequency fluctuations of the laser but also to actively stabilize the master laser frequency. Thus, in the proposed configuration there are two control loops:

- (ϕ_R − frequency offset) locks the slave laser to the master laser as in the LISA baseline
- ($\phi_F - \phi_R$ − tuning bias) locks the master laser absolute frequency.

The laser from the adjacent optical bench (the slave laser) is offset phase locked to the master with high gain/bandwidth with a constant offset:

$$\nu_s = \nu_M + f_{het} \tag{4}$$

with 2 MHz $\leqslant f_{het} \leqslant$ 20 MHz. Such a phase locking arrangement will be used in any case.

After phase locking the closed loop phase noise of the slave laser is given by (in the frequency domain)

$$P_{s|cl} = \frac{G_1}{1+G_1} P_M - \frac{G_1}{1+G_1}\epsilon_1 + \frac{1}{1+G_1} P_{s|fr}, \tag{5}$$

where

- $P_{s|fr}$—free-running slave laser phase noise
- $P_{s|cl}$—closed loop slave laser phase noise
- P_M—master laser phase noise
- ϵ_1—error point noise (sensor noise) for loop 1
- G_1—controller transfer function for loop 1 (e.g. PI controller).

Equation (5) shows that in the high gain limit,

- the free-running slave phase noise is suppressed;
- the performance is limited by sensor noise ϵ_1;
- the slave laser tracks the master laser phase noise with accuracy $\frac{G_1}{1+G_1}$ (≈1 in the high gain limit).

Offset phase locking the slave laser fixes the beatnote frequency for both the ϕ_{R} and ϕ_{F} interferometers to the chosen frequency offset, but does not have any effect on the frequency of the master laser. The master laser can be freely tuned, while the slave laser tracks these changes. Thus the first loop only controls the frequency difference between the two lasers.

2.1. Master laser frequency control loop

Measuring the phase of the beatnote for the ϕ_{F} interferometer produces the following error signal for the second control loop (used to stabilize the master laser frequency):

$$E_2 = \epsilon_2 - P_{\mathrm{M}} \exp(-\mathrm{j}\omega\tau) + P_{\mathrm{s|cl}}. \tag{6}$$

This assumes that the length (propagation delay) for the slave laser to ϕ_F is identical to the delay for ϕ_R. The master laser has an additional propagation delay of $\tau = \Delta L/c$ for ϕ_F compared to ϕ_R.

Subtracting the residual error point of slave laser control loop (E1) from the error point of the second control loop for the master laser (E2) results in the following combined error signal:

$$E_2 - E_1 = [1 - \mathrm{e}^{-\mathrm{j}\omega\tau}]P_{\mathrm{M}} + \epsilon_2 - \epsilon_1, \tag{7}$$

which is independent of the slave laser controller, simplifying the design of the controller for the master laser.

For frequencies below the inverse delay time τ^{-1} the transducer gain of the mismatched pathlength interferometer is

$$\frac{\delta\Phi_F}{\delta\nu_M} \approx \frac{2\pi\Delta L}{c} = 2\pi\tau, \tag{8}$$

thus providing an error signal which can be used to control the frequency of the master laser. The error signal for the master laser frequency is immediately available for any operating point.

The closed-loop master laser noise is then given by

$$P_{\mathrm{M|cl}} = \frac{P_{\mathrm{M|fr}}}{1 + L_2} + \frac{G_2}{1 + L_2}\epsilon_1 - \frac{G_2}{1 + L_2}\epsilon_2, \tag{9}$$

where G_2 is the controller transfer function for the master laser controller and $L_2 = G_2[1 - \exp(-\mathrm{j}\omega\tau)]$ is the loop gain.

Introducing an offset, ϵ_2, to the error point of the master laser control loop can be used to tune the master laser frequency (which the slave laser tracks due to the offset-phase lock). For large controller gain and frequencies low compared to the inverse delay time, the frequency tuning response is

$$\frac{\partial\nu_{\mathrm{M|cl}}}{\partial\epsilon_2} \approx \frac{-1}{2\pi\tau} \quad [\mathrm{Hz\ rad^{-1}}]. \tag{10}$$

2.2. Performance estimation

Like an optical cavity the performance is ultimately limited by the stability of the reference, in this case the pathlength stability of the Mach–Zehnder. Figure 3 shows the predicted system performance assuming a 50 cm pathlength mismatch and a typical free-running NPRO laser frequency noise (10 kHz $\mathrm{Hz}^{-1/2}$ at 1 Hz with $1/f$ noise shape). The assumed combined

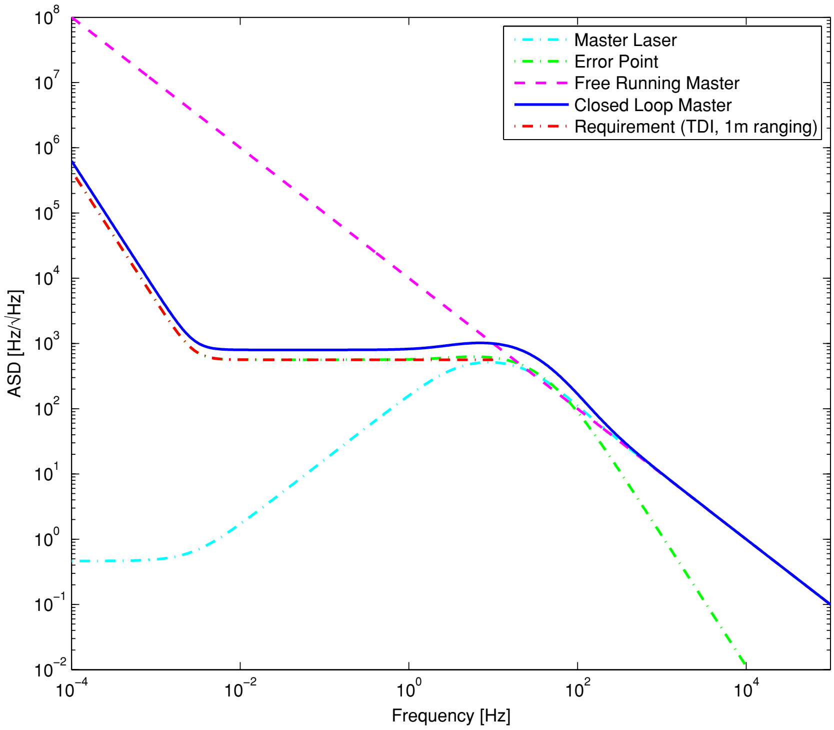

Figure 3. Closed loop frequency noise (solid blue trace) assuming a 50 cm pathlength mismatch.

phasemeter and pathlength noise is

$$\epsilon_i = \frac{2\pi}{\lambda} \times 1 \text{ pm Hz}^{-1/2} \times \sqrt{1 + \left(\frac{2.8 \text{ mHz}}{f \text{ Hz}}\right)^4}. \quad (11)$$

The phasemeter/pathlength noise of the two channels is assumed to be uncorrelated.

The closed loop frequency noise level intersects the free-running noise level at approximately 10 Hz for typical Nd:YAG NPRO lasers. For closed loop bandwidths above this frequency, the closed loop noise level would be higher than the free-running laser which has a potential impact on the performance of other subsystems (e.g. phasemeter). Therefore in the proposed design the bandwidth of the second loop is restricted to approximately 20 Hz.

In this simple model the closed loop frequency is limited primarily by the phasemeter noise of the two phasemeter channels used and results in a closed loop frequency noise level for the master laser of approximately 800 Hz $\text{Hz}^{-1/2}$ in the 10 mHz to 1 Hz range.

The performance shown in figure 3 almost meets the 4 pm $\text{Hz}^{-1/2}$ requirement for 1 m ranging accuracy (2 m effective arm length difference) *even without* arm locking. In combination with arm locking the frequency noise is several orders of magnitude below the requirement [17]. The performance could potentially be improved by increasing the arm length mismatch or by improving the phasemeter performance.

2.3. Frequency tunability

Figure 4 shows the frequency tuning response for injecting offsets into the master laser frequency control loop (solid blue curve). Note that for this input the bandwidth is restricted

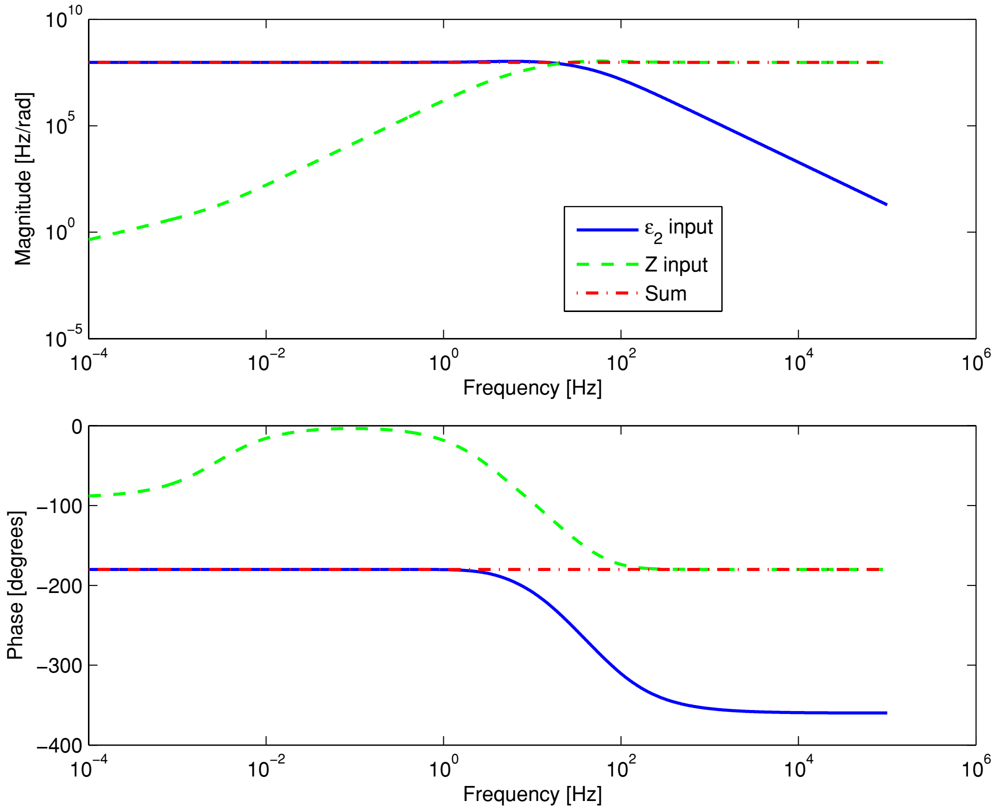

Figure 4. Frequency tuning response.

by the limited loop bandwidth used in order to reduce the degradation of the laser noise above 10 Hz. The dashed green curve shows the frequency tuning response for the input labelled Z in figure 2, scaled by a factor of $1/(2\pi\tau)$ in order to compensate for the gain of the interferometer. The sum is shown as the dashed red curve which is flat. Thus injecting the same signal into the offset and directly to the laser (compensating for the interferometer gain) provides a high bandwidth frequency actuation for implementing arm locking. The achievable tuning bandwidth with this approach is limited by the laser frequency actuators and processing delay of the phasemeter and controller electronics.

3. Conclusion

An analysis of an LTP-style unequal arm length Mach–Zehnder interferometer as an alternative frequency pre-stabilization system for LISA was presented. The ultimate performance of the technique with LISA-like hardware remains to be experimentally verified.

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