

OSCAR a Matlab based optical FFT code

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Abstract.

Optical simulation softwares are essential tools for designing and commissioning laser interferometers. This article aims to introduce OSCAR, a Matlab based FFT code, to the experimentalist community. OSCAR (Optical Simulation Containing Ansys Results) is used to simulate the steady state electric fields in optical cavities with realistic mirrors. The main advantage of OSCAR over other similar packages is the simplicity of its code requiring only a short time to master. As a result, even for a beginner, it is relatively easy to modify OSCAR to suit other specific purposes. OSCAR includes an extensive manual and numerous detailed examples such as simulating thermal aberration, calculating cavity eigen modes and diffraction loss, simulating flat beam cavities and three mirror ring cavities. An example is also provided about how to run OSCAR on the GPU of modern graphic cards instead of the CPU, making the simulation up to 20 times faster.

1. Overview of OSCAR

OSCAR is a FFT code which is able to simulate Fabry Perot cavities with arbitrary mirror profiles. One of the key features of OSCAR is the possibility to easily modify the code to tailor specific simulation purposes. OSCAR is written with the Matlab scripting language, one can import/export files (mirror maps or cavity eigen modes profile for example), create a ring cavity, create batch file or plot 2D optical field with little programming skill. The core of the code is only 400 lines long (including comments) and the manual provides five detailed examples.

The first version of OSCAR was written with the software IGOR[1] in 2005. This code was then translated to Matlab and used to calculate diffraction losses by Pablo Barriga[2]. Finally, the Matlab code has been rewritten to decrease the computational time and to add new functionality.

OSCAR is mainly intended for people who want to quickly simulate only one cavity with non Gaussian fundamental eigenmodes or input beam. The code and the manual can also be used as an educational tool to understand how internally FFT code works.

1.1. Possible simulations

OSCAR is a versatile tool to simulate Fabry Perot cavities. The following is a (non-exhaustive) list of the results which can be obtained with OSCAR:

- calculate the Gouy phase shift between higher order optical modes. It may be useful for flat beams for example, where no analytical calculations of the Gouy phase shift has been derived yet

- calculate the coupling loss between the input beam and the cavity eigen modes in the case of mode mismatching
- calculate the circulating beam (intensity and profile) for stable and also unstable cavities
- calculate diffraction loss and eigen modes of a cavity with arbitrary mirror profiles and imperfect optics.

1.2. Restrictions

OSCAR is designed to simulate anything which can be derived from the steady state, classical, optical field circulating inside a Fabry Perot cavity. It means OSCAR does not take into account radiation pressure or quantum effects. OSCAR (in the present version) can not simulate coupled cavities. For more complex simulation other FFT codes such as SIS[3] or darkF[4] exist.

2. Principle

In this section, we briefly introduce the concept of optical simulations using the Fourier transform. Unfortunately, due to the limited length of this article, no demonstration is included but the justification can be found in the references or in the OSCAR manual.

2.1. Propagation of an arbitrary optical field

It is possible to propagate any arbitrary coherent optical field under the paraxial approximation by the use of a Fourier transform. Typically such an operation requires 3 steps [5]:

- (i) Decomposition of the complex amplitude of the electric field into a sum of elementary plane waves. Mathematically, this step is achieved by a 2D Fourier transformation.
- (ii) Propagation of each plane wave, which is equivalent to adding a phase shift in the frequency domain. The phase shift depends of the distance of propagation and the spatial frequency of the plane wave.
- (iii) Recomposition of the electric field from the propagated plane waves. This step is in fact a 2D inverse Fourier transformation.

The above 3 steps make up the basis of optical FFT codes. The pseudo code shown here allows the propagation in free space of any arbitrary optical field, independently of any optical basis (Hermite or Laguerre Gauss) or assumption on the beam shape. In optical FFT codes, only the propagation requires a transformation into the spatial frequency domain, all the other operations (e.g. reflection by a mirror or transmission through an aperture) are performed directly on the complex electric field.

2.2. Adding realistic optics

The reflection by a mirror or the transmission through a lens can be described as a change in the optical field wavefront. For example, we can consider an input laser field E_i passing through an element inducing a wavefront distortion characterized by the optical path $\Delta OP(x, y)$. In this case, the transmitted field E_t can be written as[6]:

$$E_t(x, y) = E_i(x, y) \times \exp(-jk\Delta OPL(x, y)) \quad (1)$$

with k the constant of propagation. An aperture used to represent finite size mirrors can also be easily implemented by a 2D transmission matrix A . Practically, an aperture $A(x, y)$ is represented by a matrix of zeros and ones. A 0 at the position (x, y) indicates that the light is blocked (falls outside the mirror) and a 1 indicates that the light is fully reflected or transmitted.

So once again, numerically, the reflection or transmission through a finite size mirror can be described as:

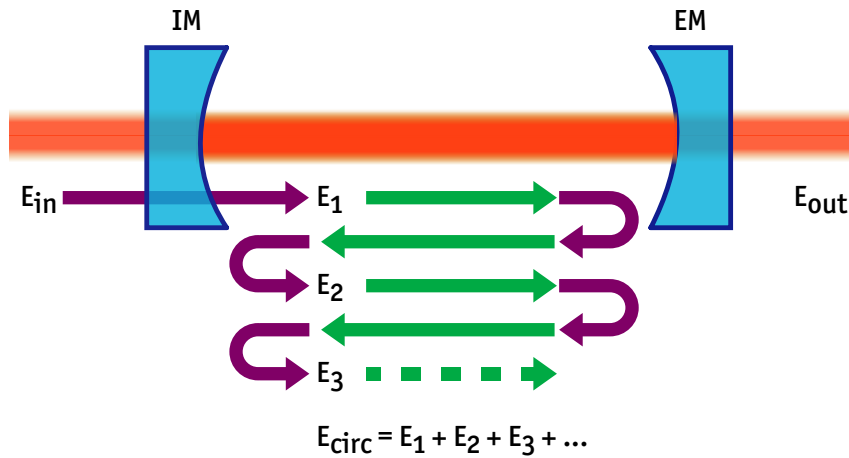


Figure 1. Description of the algorithm used in OSCAR to calculate the circulating field in a Fabry Perot cavity. The violet arrows represent a change in phase for the light field which is described in section 2.1 and the green arrows represent the propagation of the light field using a FFT code. From E_i to E_{i+1} , the light field has undergone one round trip in the cavity. IM and EM stand respectively for Input Mirror and End Mirror. E_{circ} is the circulating power inside the cavity.

$$E_o(x, y) = E_i(x, y) \times \exp(-jk\Delta OPL(x, y)) \times A(x, y) \quad (2)$$

2.3. Simulating a Fabry-Perot cavity

A Fabry-Perot cavity is made up of two mirrors facing each other. Between these two mirrors, a light field is circulating, bouncing back and forth between the two reflective coatings. One of the main interest of the Fabry-Perot cavity is that the optical power of the circulating field can be much higher than the power of the input field. With OSCAR it is possible, for a given input field, to calculate the total circulating power, reflected power and transmitted power as well as the spatial profile of all the light fields.

OSCAR calculates the circulating field by propagating back and forth the laser beam between the two mirrors and then summing all the transient fields at one reference plane [7] as shown in figure 1. This method can also be used to analytically calculate the circulating power in a Fabry Perot cavity [8].

In more detail, we can write the OSCAR algorithm used to compute the circulating power. Using the notation from the figure 1 the different consecutive steps can be described as:

- (i) Define the cavity parameters as well as the mirror profiles and the input beam.
- (ii) Propagate the input beam E_{in} through the input mirror. For this purpose the input mirror can be exchanged with a lens, so the equation (2) can be used. As a result, we obtain the field E_1 .
- (iii) After one round trip in the cavity, the field E_1 becomes E_2 . One round trip in the cavity consists specifically of one propagation through the cavity length using the FFT code, one reflection on the end mirror, another propagation back to the input mirror and then finally a reflection on the input mirror.
- (iv) Repeat the last operation to create the set of electric field $\{E_i\}$.

- (v) Then sum all the field E_i to have the cavity circulating power E_{circ} . The number of light field E_i to be considered to have an accurate result depends on the finesse of the cavity.
- (vi) The transmitted field E_{out} is simply the circulating field transmitted through the ETM.

In this above pseudo-code, we did not mention any resonance condition to maximize the circulating power in the cavity. Practically, we should always define the round trip phase shift for the field in the cavity (or a microscopic position shift for the cavity length) before calculating the cavity circulating power. The round trip phase shift allows us to set the cavity to be resonant for the fundamental mode or any other optical modes if necessary.

3. Applications

The OSCAR package is provided with several detailed examples to show the different possible simulations that can be carried out using optical FFT codes. In this section, we will focus on two examples: how to simulate flat beam cavities and how to calculate diffraction loss.

3.1. Coupling loss for flat beams cavity

In this example, we are interested in calculating the coupling loss (also called mode matching loss) between an input Gaussian beam and a Fabry-Perot cavity supporting flat (or mesa) beams[9]. Since flat beams exhibit lower power density compared to fundamental Gaussian beams, they have often been presented as a possible way to reduce the mirror thermal noise level[10] as well as the amplitude of thermal lensing.

It is possible to directly import the mirror profiles to support flat beam into OSCAR, profiles which are usually derived using the relatively complicated analytical formula. A simpler approach can be implemented: the mirror profiles are given by the wavefront curvature of the flat beam at the mirror position. Since the flat beam is an eigen mode of the cavity, the curvature of the mirrors must match the wavefront of the incoming beam.

For this example, the length L of the cavity simulated is 2 km. The power transmission of the input mirror is 0.5 %, 50 ppm for the end mirror, and the loss per reflection on each mirror is also 50 ppm. The input beam is a Gaussian fundamental mode with a beam radius of 4 cm and a wavefront curvature of 1 km. The input power is 1 W and the wavelength λ is 1064 nm. The mirror profiles are symmetric and created to support a concentric flat beam of width $b = 2w_0$ with $w_0 = \sqrt{L\lambda/2\pi}$ following the notation from[11].

For the above setup, the steady state circulating power can be calculated with OSCAR and is found to be 546 W. Now, to calculate the coupling loss, the circulating power has to be compared to the circulating power in the case of perfect mode matching. That can be done analytically in this simple case or it can also be done in OSCAR by replacing the input beam by the beam used to create the mirror profile. The circulating power in case of perfect mode matching is 752 W, so for the simulated setup the coupling loss is around 27 %¹.

3.2. Diffraction loss inside a mode cleaner

One of the most promising application of FFT optical codes is to calculate the diffraction loss of the circulating cavity field (or cavity eigen mode). As a reminder, the diffraction loss is the loss due to the finite size of the mirrors. This loss is also sometime referred to as clipping loss. Since in theory, Gaussian beams have an infinite spatial extent, diffraction losses are always present. However the loss can become negligible for large diameter mirrors and small laser beam radius.

The optical setup under consideration is a 3 mirror ring cavity with the input and output mirrors flat and the third mirror curved. The distance between the 2 flat mirrors is 6 cm and

¹ It is just a numerical example, the coupling loss between input Gaussian beam and flat beam cavity can be much lower

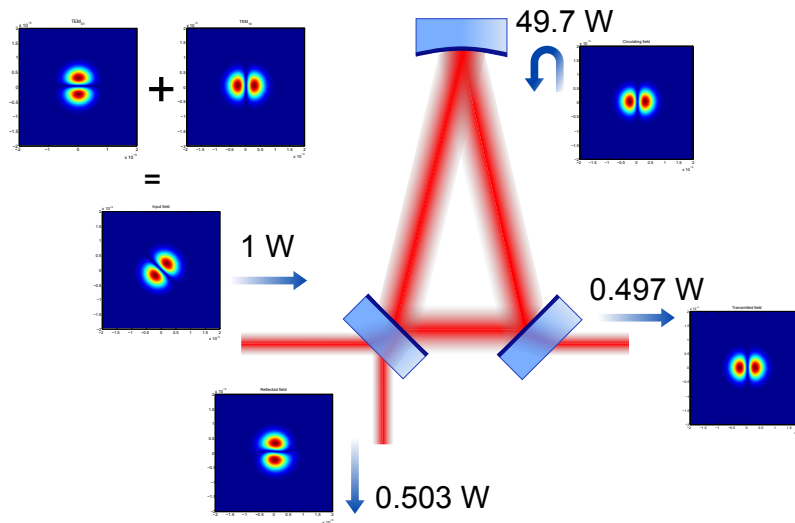


Figure 2. Optical setup of the mode cleaner with the input beam coming from the left side. The optical power in the different fields is also mentioned.

the distance between the curved mirror and the optical segment between the 2 flat mirrors is 20 cm. The radius of curvature of the curved mirror is 1 m, giving a g-factor of 0.77 and a waist size of 0.38 mm located between the flat mirrors. The finesse of the cavity is 310. All the mirrors have a diameter of 3 mm (this value is not realistic but interesting for the simulations).

To make the experiment more interesting, the input beam could be the sum of a TEM_{10} and a TEM_{01} as shown in figure 2. Both beams of power 0.5 W are well matched to the cavity (but not perfectly matched).

We select the mode cleaner to be resonant for the mode TEM_{10} and as a result the mode TEM_{01} is reflected. The diffraction loss is derived by calculating the round trip loss of the circulating mode when the reflectivities of all the cavity mirrors are set to 1. In this manner, the diffraction loss is found to be 15 ppm per round trip.

4. Calculation on the GPU

Graphical Process Unit (GPU) have recently gained special interest in the scientific community thanks to their parallel computing ability in conjunction with the availability of high level programming languages. OSCAR has also been adapted to run its most intensive calculations on the GPU using the Matlab library called GPUmat[12]. GPUmat is freeware and it requires the CUDA[13] toolkit to be installed². The GPU library allows the transfer of variables from the CPU to the GPU and overloads common Matlab operators and functions which can then be executed directly on the GPU without any need of another programming language.

To test the GPU capability, we ran OSCAR on a dedicated machine with an Intel Quad core CPU (Q8200) with 8 Gb of RAM. The GPU is housed within a Telsa C1060 graphic card with 4 Gb of memory. On the software end, the operating system is Linux Ubuntu 9.04 and we use Matlab2009a, with GPUmat version 0.1 and the CUDA drivers version 2.3.

The comparison in execution time between the CPU and GPU for 100 light round trips in a cavity is shown in figure 3. For large size matrices, the GPU is more than 20 times faster than the CPU. However, to be fair, it should be mentioned that the calculation on the GPU is done with single precision numbers whereas on the CPU it is done with double precision (even

² so it only works with modern Nvidia graphic cards

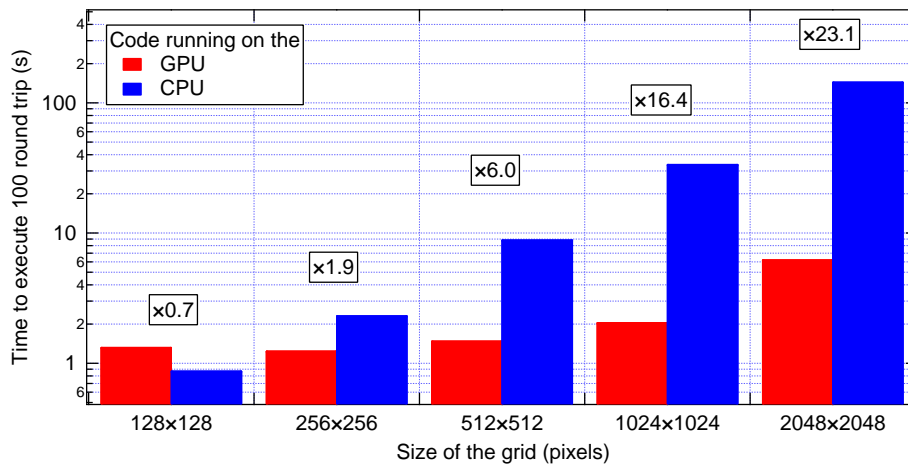


Figure 3. Speed comparison for the same code running on the CPU (blue bars) and on the GPU (red bars). The number in the box, indicates the gain factor of running OSCAR on the GPU. For small grid size, the GPU is slower since a large amount of time is taken by the transfer of variables between the CPU and the GPU memory.

if the variable is declared as a single in Matlab). The use of single precision or double precision number does not bring any difference in the results from OSCAR.

Regarding the future of FFT codes on the GPU, we can already take note that recent GPU cards support double precision numbers (at the price of a slower execution time) and using the Matlab Parallel Computing Toolbox, it is possible to run a single program across several GPU cards[14].

5. Conclusion and outlook

In this article we have presented OSCAR a versatile tool to simulate steady state Fabry-Perot cavity with imperfect mirrors. OSCAR is written for Matlab and special attention was drawn to make the code clear and easily modifiable. Moreover a fully documented manual is provided with examples.

In the next versions of OSCAR coupling cavities will be implemented and ultimately a dual recycling interferometers with Fabry-Perot arm cavities could be simulated. Interested users may download OSCAR from two websites quoted in references [15, 16].

References

- [1] IGOR software <http://www.wavemetrics.com/> 1
- [2] Barriga P, Bhawal B, Ju L and Blair D G 2007 *J. Opt. Soc. Am. A* **24** 1731–1741 1
- [3] Yamamoto H SIS talk at LMA <https://dcc.ligo.org/cgi-bin/private/DocDB/ShowDocument?docid=58712> 2
- [4] Laval M Presentation of DarkF http://wwcascina.virgo.infn.it/commissioning/OPC/Talks/DarkFstatus_08_03_2006.ppt 2
- [5] Siegman A E 1986 *Lasers* (University Science Books) 2
- [6] Bochner B 1991 *Modelling the Performance of Interferometric Gravitational-Wave Detectors with Realistically Imperfect Optics* Ph.D. thesis MIT 2
- [7] Saha P 1997 *J. Opt. Soc. Am. A* **14** 2195–2202 3
- [8] Silfvast W T 2004 *Laser Fundamentals* (Cambridge University Press) see section 11.2 3
- [9] Bondarescu I M and Thorne K S 2006 *Phys. Rev. D* **74** 082003 4
- [10] Vinet J Y 2005 *Classical and Quantum Gravity* **22** 1395–1404 4

- [11] Agresti J 2007 *Researches on Non-standard Optics for Advanced Gravitational Waves Interferometers*. Ph.D. thesis University of Pisa 4
- [12] GPU mat library <http://gp-you.org/> 5
- [13] CUDA website http://www.nvidia.com/object/cuda_home.html 5
- [14] Jacket v12 for Matlab http://www.accelereyes.com/release_notes.php 6
- [15] Matlab central website <http://www.mathworks.com/matlabcentral/fileexchange/20607> 6
- [16] GEO simulation group wiki <http://www.sr.bham.ac.uk/dokuwiki/doku.php?id=geosim:oscar> 6