

Axions without Peccei-Quinn Symmetry

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Abstract

We argue that the axion arising in the solution of the strong CP problem can be identified with the Majoron, the (pseudo-)Goldstone boson of spontaneously broken lepton number symmetry. At low energies, the associated $U(1)_L$ becomes, via electroweak parity violation and neutrino mediation, indistinguishable from an axial Peccei-Quinn symmetry in relation to the strong interactions. The axionic couplings are then *fully computable* in terms of known SM parameters and the Majorana mass scale, as we illustrate by computing the effective couplings to photons and quarks at two loops.

1. Introduction. The solution of the strong CP problem by means of the Peccei-Quinn mechanism [1] is commonly assumed to require the presence of a *chiral* $U(1)_{PQ}$ symmetry (Peccei-Quinn symmetry) which is not part of the standard model (SM). When spontaneously broken, this symmetry gives rise to a (pseudo-)Goldstone boson, the *axion* [2, 3]. The latter is usually described by a pseudoscalar field transforming by constant shifts under $U(1)_{PQ}$. The absence of CP violation in the strong interactions is then explained by the fact that any contribution to the θ parameter can be absorbed into such a shift, so the problem is solved if the axion vacuum expectation value dynamically adjusts itself to zero [4]. To accommodate the extra $U(1)_{PQ}$ the available models realizing this idea invariably need to introduce (so far unobserved) new particles and scales beyond the SM, such as new heavy quarks or non-standard Higgs fields [5, 6].

In [7] a minimal extension of the SM was proposed, based on the hypothesis that quantum mechanically broken conformal symmetry stabilizes the electroweak hierarchy, with only the right-chiral neutrinos ν_R^i and one

complex scalar field

$$\phi(x) = \varphi(x) \exp\left(\frac{ia(x)}{\sqrt{2}\mu}\right) \quad (1)$$

as new ingredients [8, 9]. This field is a singlet under the SM symmetries and couples only to right-chiral neutrinos. If ϕ acquires a vacuum expectation value by (possibly radiatively induced) spontaneous symmetry breaking, a Majorana mass term is generated for the right-chiral neutrinos, such that for instance the smallness of the light neutrino masses can be naturally explained with appropriate neutrino Yukawa couplings and without the need to introduce a large Majorana mass ‘by hand’. The phase $a(x)$ then gives rise to a (pseudo-)Goldstone particle (usually called ‘Majoron’) associated with the spontaneous breaking of global $U(1)_L$ lepton number symmetry.

As argued in [7] the Majoron has several features in common with the axion, and the smallness of its couplings can be tied to the smallness of neutrino masses. In this Letter, we go one step further and propose that the Majoron actually *is* the axion, with *computable* effective couplings to SM particles, and the neutrino Yukawa couplings as the only unknown parameters (a possible link between light neutrinos and the invisible axion had already been suggested in [10]). In other words, we claim that lepton number symmetry $U(1)_L$ is transmuted, via electroweak parity violation and neutrino mixing, into a $U(1)$ symmetry that, in relation to the strong interactions, is indistinguishable from the standard axial Peccei-Quinn symmetry at low energies. We present exact expressions for the (UV finite) two-loop integrals describing the coupling of the axion to photons and (light) quarks; the main technical novelty here is the consistent use of the off-diagonal neutrino propagators (6) below. From the quark couplings one can estimate the coupling of the axion to gluons, which is naturally tiny.

Because Goldstone bosons couple only via derivatives, the (perturbative) effective action at low energies contains only terms $\propto \mathfrak{X}^\mu \partial_\mu a$, where \mathfrak{X}^μ are local expressions in the SM quantum fields. At lowest order there are only three possible candidates for \mathfrak{X}^μ : (i) a Chern-Simons current, which by partial integration is equivalent to a coupling $a \text{Tr} W_{\mu\nu} \widetilde{W}^{\mu\nu}$ (where W_μ can be any SM gauge connection), (ii) a vector current \mathcal{J}_V^μ and (iii) an axial current \mathcal{J}_A^μ . Being mediated by the weak interactions the fermionic bilinears contributing to \mathfrak{X}^μ and involving charged SM fermions all appear in ‘V – A’ form. Therefore, whenever $\partial_\mu \mathcal{J}_V^\mu \approx 0$ by some (approximate) conservation law, $a(x)$ couples like a *pseudoscalar*. This is the case for all relevant axionic

couplings, in particular those involving photons, gluons or electrons.

2. Lagrangian. We refer to [11, 12] for basic properties of the SM, and here only quote the Yukawa couplings

$$\begin{aligned} \mathfrak{L}_{\text{int}} = & \left(\overline{L}^i \Phi Y_{ij}^E E^j + \overline{Q}^i \Phi Y_{ij}^D D^j + \overline{Q}^i \varepsilon \Phi^* Y_{ij}^U U^j \right. \\ & \left. + \overline{L}^i \varepsilon \Phi^* Y_{ij}^\nu N^j + \phi N^{iT} \mathcal{C}^{-1} Y_{ij}^M N^j + \text{h.c.} \right) \end{aligned} \quad (2)$$

and the neutrino terms in the Lagrangian, see (4) below. Here Q^i and L^i are the left-chiral quark and lepton doublets, U^i and D^i the right-chiral up- and down-like quarks, while E^i are the right-chiral electron-like leptons, and $N^i \equiv \nu_R^i$ the right-chiral neutrinos (we suppress all indices except the family indices $i, j = 1, 2, 3$). Φ is the usual Higgs doublet, and ϕ is the new complex scalar field introduced in (1). As is well known [11, 12], one can use global redefinitions of the fermion fields to transform the Yukawa matrices Y_{ij}^E , Y_{ij}^U and Y_{ij}^M to real diagonal matrices. By contrast, the matrices Y_{ij}^D and Y_{ij}^ν may exhibit (strong) mixing. Besides the standard (local) $SU(3)_c \times SU(2)_w \times U(1)_Y$ symmetries, the Lagrangian (2) admits two *global* $U(1)$ symmetries, baryon number symmetry $U(1)_B$ and lepton number symmetry $U(1)_L$. The latter is associated with the Noether current

$$\mathcal{J}_L^\mu := \overline{L}^i \gamma^\mu L^i + \overline{E}^i \gamma^\mu E^i + \overline{N}^i \gamma^\mu N^i - 2i \phi^\dagger \overleftrightarrow{\partial}^\mu \phi \quad (3)$$

The fact that ϕ carries lepton charge is crucial for the effect to be discussed below, namely the proposed transmutation of $U(1)_L$ into a Peccei-Quinn-like symmetry.

For the computation of loop diagrams it is convenient to employ $SL(2, \mathbb{C})$ spinors [7]. With $\nu_L^i \equiv \frac{1}{2}(1 - \gamma^5)\nu^i \equiv \bar{\nu}^{i\dot{\alpha}}$ and $\nu_R^i \equiv \frac{1}{2}(1 + \gamma^5)\nu^i \equiv N_\alpha^i$, the neutrino part of the free Lagrangian reads (see [13] for conventions)

$$\begin{aligned} \mathfrak{L} = & \frac{i}{2} \left(\nu^{i\alpha} \not{\partial}_{\alpha\dot{\beta}} \bar{\nu}^{i\dot{\beta}} + N^{i\alpha} \not{\partial}_{\alpha\dot{\beta}} \bar{N}^{i\dot{\beta}} \right) + \\ & m_{ij} \nu^{i\alpha} N_\alpha^j + \frac{1}{2} M_{ij} N^{i\alpha} N_\alpha^j + \text{c.c.} \end{aligned} \quad (4)$$

with the (complex) Dirac and Majorana mass matrices $m_{ij} = Y_{ij}^\nu \langle H \rangle$ and $M_{ij} = Y_{ij}^M \langle \varphi \rangle$, respectively (where $\langle H \rangle^2 \equiv \langle \Phi^\dagger \Phi \rangle$). Rather than diagonalize the fields w.r.t these mass terms, we work with *non-diagonal propagators* and

with the interaction vertices from (2). Defining

$$\mathcal{D}(p) := \left[p^4 - p^2(M^{-1}m^T m^* M + m^\dagger m + M^* M) + m^\dagger m M^{-1} m^T m^* M \right]^{-1} \quad (5)$$

we obtain the matrix propagators (in momentum space)

$$\begin{aligned} \langle \nu_\alpha^i \nu_\beta^j \rangle &= i [m^* M \mathcal{D}(p) m^\dagger]^{ij} \varepsilon_{\alpha\beta} \\ \langle \nu_\alpha^i \bar{\nu}_\beta^j \rangle &= i [(m^T)^{-1} \{p^2 - M M^* - (M^*)^{-1} m^\dagger m M^*\} \mathcal{D}(p)^* m^T]^{ij} \not{p}_{\alpha\beta} \\ \langle N_\alpha^i N_\beta^j \rangle &= i [M^* p^2 \mathcal{D}(p)^*]^{ij} \varepsilon_{\alpha\beta} \\ \langle N_\alpha^i \bar{N}_\beta^j \rangle &= i [(p^2 - M^{-1} m^T m^* M) \mathcal{D}(p)]^{ij} \not{p}_{\alpha\beta} \\ \langle \nu_\alpha^i N_\beta^j \rangle &= i [m^* \{p^2 - (M^*)^{-1} m^\dagger m M^*\} \mathcal{D}(p)^*]^{ij} \varepsilon_{\alpha\beta} \\ \langle \nu_\alpha^i \bar{N}_\beta^j \rangle &= -i [m^* M \mathcal{D}(p)]^{ij} \not{p}_{\alpha\beta}, \end{aligned} \quad (6)$$

together with their complex conjugate components. Evidently, these propagators allow for maximal mixing in the sense that every neutrino component can oscillate into any other (also across families). For the UV finiteness of the diagrams to be computed below it is essential that some of the propagator components fall off like $\sim p^{-3}$, unlike the standard Dirac propagator. Taking M_{ij} diagonal it is not difficult to recover the mass eigenvalues as predicted by the standard seesaw formula [14, 15, 16].

With the above propagators and the (extended) SM Lagrangian we can now proceed to compute various effective low energy couplings involving the ‘axion’ a which are mediated by neutrino mixing via two or three-loop diagrams. Here we present only the results for photon-axion and quark-axion couplings, cf. the diagrams depicted below. Further results and detailed derivations will be given in a forthcoming publication [17].

3. Photon-axion vertex. For the low energy effective action we need only retain contributions where all particles circulating inside the loops are much heavier than the external particles. As our first example we determine the effective coupling of the axion to photons via the two-loop diagram in Fig. 1. For small axion momentum $q^\mu = k_1^\mu - k_2^\mu$ it is possible to derive a closed

form expression for the two-loop integral and for arbitrary mixing matrices [17]. Setting $\mu = \langle\varphi\rangle$ in (1) and denoting by M_j the eigenvalues of the (diagonal) matrix M_{ij} , a lengthy calculation gives the expected kinematical factor $\epsilon^{\mu\nu\lambda\rho}k_{1\lambda}k_{2\rho}$ with coefficient function

$$\begin{aligned} & \frac{i e^2 g_w^2}{64\sqrt{2}\pi^4} \sum_{i,j} \frac{|m_{ij}|^2 M_j^2}{\langle\varphi\rangle} \int_0^1 dx \int_0^1 dy \int_0^1 dz \int_0^1 dt \times \\ & x(1-x)y^2z(1-z)t^3 \left\{ \frac{-yt - 6(1-y)zt}{\mathbb{M}_{ij}^4(x,y,z,t)} + \right. \\ & \left. \frac{y(1-y)[-2y - 3(1-y)zt][(2-t)m_{e_i}^2 - t(1-t)^2k_2^2]}{\mathbb{M}_{ij}^6(x,y,z,t)} \right\} \end{aligned} \quad (7)$$

where $m_{e_i} \equiv (m_e, m_\mu, m_\tau)$ and

$$\begin{aligned} \mathbb{M}_{ij}^2(x,y,z,t) & := xyzzM_j^2 + (1-y) \left[ztM_W^2 + \right. \\ & \left. -yt(1-t)k_2^2 + y(1-zt)m_{e_i}^2 \right] \end{aligned} \quad (8)$$

The above integral is cumbersome to evaluate in general form, but for small photon momenta $k_1^\mu \approx k_2^\mu$ we get

$$(7) \approx \frac{i\alpha_{em}\alpha_w}{72\sqrt{2}\pi^2} \sum_{i,j} \frac{m_{ij}^2}{\langle\varphi\rangle M_j^2} \left(\log \frac{M_j^2}{m_e^2} \right)^2 \quad (9)$$

Of course, the precise value of the effective low energy coupling depends on the (unknown) values of the Yukawa mass matrices m_{ij} and $M_{ij} = M_j\delta_{ij}$. For an estimate we take $M_j \sim M$, and assume the matrix entries m_{ij} to be of the same order of magnitude $\sim m$ (strong mixing). For the effective axion-photon vertex we thus obtain

$$\mathfrak{L}_{\text{eff}}^{a\gamma\gamma} = \frac{1}{4f_a} a F^{\mu\nu} \tilde{F}_{\mu\nu}, \quad f_a^{-1} = \frac{\alpha_{em}\alpha_w \sum m_\nu}{72\sqrt{2}\pi^2 M^2} \left(\log \frac{M^2}{m_e^2} \right)^2 \quad (10)$$

with the usual seesaw relation $\sum m_\nu \sim \sum m^2/M$. Substituting numbers we find $f_a = \mathcal{O}(10^{16} \text{ GeV})$ which is outside the range of existing or planned experiments [18]. Thus the smallness of axion couplings gets directly tied to the smallness of the light neutrino masses via (10).

4. Quarks and gluons. The effective low energy couplings to light quarks can be analyzed in a similar way. With $P_L \equiv \frac{1}{2}(1 - \gamma^5)$ we parametrize these couplings as

$$\mathfrak{L}_{\text{eff}}^{aqq} = i \partial_\mu a \left(c_{ij}^{aUU} \bar{U}^i \gamma^\mu P_L U^j + c_{ij}^{aDD} \bar{D}^i \gamma^\mu P_L D^j \right) \quad (11)$$

Again one can obtain an exact formula for the (UV finite) two-loop integrals; e.g. for the up-like quarks we get

$$c_{ij}^{aUU} = \sum_{k,r,s} \frac{g_w^4 |m_{rs}|^2 M_s^2 V^{ik} (V^\dagger)^{kj}}{128 \sqrt{2} \pi^4 \langle \varphi \rangle} \times \quad (12)$$

$$\int_0^1 dx \int_0^1 dy \int_0^1 dt \int_0^1 dz x(1-x)y^3(1-z)t^3 \times$$

$$\frac{-1 + 3y + 3(1-y)zt}{[xyztM_s^2 + (1-y)\{yt(1-z)M_W^2 + ztm_{e_r}^2 + y(1-t)m_{D_k}^2\}]^2}$$

with the CKM matrix V^{ij} . A similar (but not the same) formula is obtained for c_{ij}^{aDD} [17]. In principle, there are also contributions from diagrams with Z -boson exchange, but these can be disregarded for the effective low energy Lagrangian because they involve a purely neutrino triangle with one light neutrino (which is lighter than any external quark). To estimate the integral, we set $m_{e_i} = m_{D_i} = 0$ in (12) (which still leaves a convergent integral that can be calculated exactly [17]). Because the CKM matrix is unitary, both c_{ij}^{aUU} and c_{ij}^{aDD} become proportional to δ_{ij} to leading order, and we obtain

$$c_{ij}^{aUU} = \sum_{k,l} \frac{\alpha_w^2 |m_{kl}|^2}{128 \sqrt{2} \pi^2 \langle \varphi \rangle M_j^2} \left[\left(\log \frac{M_j^2}{M_W^2} - 2 \right)^2 + \frac{2\pi^2}{3} \right] \delta_{ij} \quad (13)$$

assuming $M_j > M_W$; if instead we take $M_j = M_W$ the exact result replaces the square bracket by 0.71 (note that the Majorana mass M is much closer to the weak scale in [8, 7] than in the usual see-saw scenario).

By the approximate (flavor-wise) conservation of the up and down quark vector currents, we can now drop the vectorlike contribution in the effective Lagrangian which thus becomes purely axial to leading order, *viz.*

$$\mathfrak{L}_{\text{eff}}^{aqq} \rightarrow i \partial_\mu a \left(g_{aUU}^{-1} \bar{U}^j \gamma^5 \gamma^\mu U^j + g_{aDD}^{-1} \bar{D}^j \gamma^5 \gamma^\mu D^j \right) \quad (14)$$

At subleading order off-diagonal contributions to c_{ij}^{aUU} and c_{ij}^{aDD} will appear with both vector and axial vector interactions. The numerical values of the effective coupling constants can be read off from the above results. Their precise values are subject to the same caveats as mentioned before (10). With the same assumptions on the Yukawa mass matrices as for (10) we get

$$g_a^{-1} \equiv g_{aUU}^{-1} \sim g_{aDD}^{-1} \sim \mathcal{O}(10^{-3}) \frac{\alpha_w^2 \sum m_\nu}{M^2} \quad (15)$$

If M is not very much larger than the weak scale M_W , we get $g_{aUU} \sim 10^{18}$ GeV for $\sum m_\nu \approx 1$ eV.

The axion-gluon coupling involves various three-loop diagrams, now with all six quarks in the loop [7]. We can shortcut this calculation by integrating the effective vertex (14) by parts, using the anomalous conservation of the axial (color singlet) quark current (see e.g. [19])

$$\partial_\mu (i \bar{q} \gamma^5 \gamma^\mu q) = \frac{\alpha_s}{4\pi} \text{Tr} G^{\mu\nu} \tilde{G}_{\mu\nu} \equiv \mathcal{Q} \quad (16)$$

with the gluonic topological density $\mathcal{Q}(x)$ (in principle there could appear extra terms $\propto m_q \bar{q} \gamma^5 q$ on the r.h.s., but Goldstone's Theorem assures us that such non-derivative terms must drop out in the final result (17)). Summing over the six quark flavors we thus obtain

$$\mathcal{L}_{\text{eff}}^{agg} = \frac{6\alpha_s}{4\pi g_a} a \text{Tr} G^{\mu\nu} \tilde{G}_{\mu\nu} \equiv 6g_a^{-1} a \mathcal{Q} \quad (17)$$

5. Axion potential. Being a Goldstone boson, the axion cannot acquire a mass in perturbation theory; likewise its vacuum expectation value remains undetermined in perturbation theory. However, non-perturbative effects can generate a potential for the axion and thereby lift the vacuum degeneracy. Using the formula $\langle \exp F \rangle = \exp [\langle F \rangle + \frac{1}{2} (\langle F^2 \rangle - \langle F \rangle^2) + \dots]$ together with $\langle \mathcal{Q}(x) \rangle = 0$ (where $\langle \dots \rangle$ denotes the gluon field average), the axion potential takes the form

$$V_{\text{axion}}(a) = \frac{1}{2} m_a a^2 + \mathcal{O}(a^3) \quad (18)$$

with the axion mass

$$m_a = 6g_a^{-1} \left[\int d^4x \langle \mathcal{Q}(x) \mathcal{Q}(0) \rangle \right]^{\frac{1}{2}} \sim 6g_a^{-1} \Lambda_{QCD}^2 \quad (19)$$

We conclude that (1) indeed $\langle a \rangle = 0$ as required for the solution of the strong CP problem, and (2) an axion mass term can be generated by non-perturbative effects. The above formulas yield the estimate $m_a \sim 10^{-9}$ eV, which may be still compatible with the axion being a (cold) dark matter candidate, at least according to standard reasoning [20, 21], and bearing in mind the considerable uncertainties in these numbers. From (15) it is evident that the viability of this dark matter scenario requires the Majorana scale M to be not much larger than M_W , in contrast to the standard see-saw proposal [14, 15, 16]. This is a main new feature of the present proposal: if true, it could be interpreted as additional evidence for a hidden conformal symmetry of the SM [7, 9], such that the observed diversity of scales in particle physics could be explained via quantum mechanically (or quantum gravitationally) induced logarithmic effects [22].

The main virtue of the present proposal is that it provides a *single* source of explanation for axion couplings and neutrino masses, tying together in a most economical manner features of the SM previously thought to be unrelated. Given the known SM parameters, and parametrizing the unknown physics in terms of just the Yukawa mass matrices, all relevant couplings are entirely calculable in terms of UV finite diagrams, and *naturally* come out to be *very small* without the need for any fine tuning.

Finally, we note that all results in this Letter can be equivalently obtained if we take the scalar field $\phi(x)$ in (1) to be *real*, absorbing the phase $a(x)$ into a redefinition of the lepton fields. This point will be discussed in much more detail in [17]. The redefinition also shows that the apparent periodicity of $a(x)$ in (1) is spurious because the redefined Lagrangian involves the field $a(x)$ only through its derivatives. Rather, the periodicity parameter for a is set by the effective action (17) and the fact that the gluon term is a topological density (see e.g. [23]).

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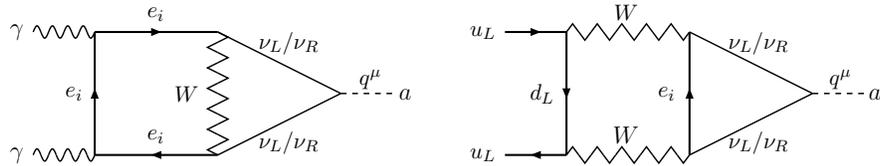


Fig.1. Axion-photon-photon and axion-quark-quark effective couplings