PROGRESS IN GRAVITATIONAL WAVE DETECTION

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Abstract

This paper reviews recent developments in our understanding of gravitational wave sources and gives a progress report on the analysis of data taken during a 100-hour coincidence run between the Glasgow and Garching prototype interferometric detectors. Recent numerical calculations of gravitational waves from supernovae and from coalescing binaries are reviewed, recently proposed sources of a cosmological background of gravitational radiation are mentioned, and our present expectations regarding gravitational wave instabilities in neutron stars are discussed. The "100-hour run" data provide an excellent opportunity to discover the complexities of analyzing real data, and have allowed our group at Cardiff to develop a prototype automatic data analysis system.

1 Introduction

Proposals to build large-scale interferometric detectors for gravitational waves are now well advanced in a number of countries. The LIGO project in the USA is being considered right now in Congress, and European plans for VIRGO and GEO detectors are also with the funding agenices of Italy, France, Germany, and Britain. Within five years it is likely that the first observations will be made with a sensitivity of $h \sim 10^{-21}$, and a further factor of 10 improvement may follow within the next 5 years. It is important to try to predict what likely sources will be seen, not only

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[*] We would like to dedicate this article to the memory of Brian Meers, whose untimely death just as this paper was being finished (January 1992) has deprived the field of one of its most creative thinkers.

in order to justify the considerable expense of building detectors but also in order to design suitable data analysis methods. For the foreseeable future, the principal barrier to detection will be the internal noise of the detectors, and therefore any information about sources that allows one to do pattern matching in order to dig into the detector noise will be crucial to the success of these instruments. Moreover, gravitational waves with a detectable strength will probably be rather rare, so the more we know about what to expect about incoming gravitational waves, the easier it will be to distinguish them with confidence from rare and poorly understood internal disturbances in detectors.

Gravitational wave sources have been reviewed recently and comprehensively in a number of places[1, 2, 3], so we will confine ourselves here to updating the ways in which recent calculations have changed our thinking about sources in the last couple of years. After considering supernovae, coalescing binaries, the stochastic background, and Wagoner stars in turn, we will then describe the work we are doing at Cardiff to design an automatic data analysis system and apply it to data taken by the two prototypes in Glasgow and Garching.

2 Update on Sources

2.1 Supernovae

Although supernova explosions were the original motivation for the development of gravitational wave detectors, they remain one of the most uncertain sources. We have a pretty good idea how many gravitational collapses occur in a given galaxy or cluster (perhaps once a week among the 2000 galaxies of the Virgo cluster), but since spherical collapse produces no radiation, we have little idea of how much radiation will come from most supernovae. It is generally felt that significant non-sphericity will arise only if the collapsing core preserves enough angular momentum to force it into non-axisymmetry.

A number of groups are preparing fully three-dimensional computer simulations of gravitational collapse with rotation, but results are still a year or two away. Realistic calculations with neutrino transport and nuclear physics are probably a decade or more away. There have been good axisymmetric collapse calculations in the last year or so,[4, 5, 6] which show, as expected, only small amounts of radiation. One typically gets only $10^{-8}M_{\odot}\,c^2$ of energy in gravitational waves, which would imply an amplitude of $h \sim 10^{-24}$ from such a collapse in Virgo. Such an event would still be detectable by second-stage interferometers if it occurred in our Galaxy, which is reassuring if one feels that axisymmetric collapse is the norm. One interesting result of these calculations is an indication that *the dominant frequency of the emitted radiation may be lower than we have assumed up to now, perhaps below 500 Hz.* This is important for the design of bar detectors (some of which observe well above 1 kHz), and especially for first-stage interferometers, where effort will be needed to reduce low-frequency sources of noise, such as seismic noise and wire suspension resonances. See the papers in this volume by Marck and Schäfer for more details.

If rotation dominates a collapse, then the classic "bar instability" of rotating stars may produce a tumbling, cigar-shaped core that emits substantial radiation. How much radiation comes out can be guessed by considering the extreme case of the radiation emitted when two neutron stars coalesce from a circular orbit. This is a case where rotation dominates the hydrodynamic support of the system, and the stars merge only because gravitational radiation carries away angular momentum. Recent landmark calculations by Nakamura and Oohara[7, 8] are described in more detail in the following section and in Nakamura's article in this volume. These suggest that up to $10^{-2} M_\odot c^2$ of energy can be carried away by gravitational waves, yielding an effective amplitude of about 10^{-21} for an event in the Virgo cluster.

These calculations also provide suggestions of what sort of waveforms we ought to look for in gravitational wave data. This is crucially important, since if we know the waveform, then the signal-to-noise ratio of the observation will depend primarily on the total energy emitted.[2] If we don't have waveform information available to us, then the signal-to-noise will depend on the peak amplitude, which could be rather low even for an energetic source if the waves come out over a relatively long time (several rotation periods of a tumbling core, for example).

On the question of whether collapse is dominated by rotation, theoretical arguments and observational data give contradictory evidence. Arguing against rotation-dominated collapse is the observation that young pulsars seem to be rather slowly rotating: the only rapid rotators are recycled pulsars spun up by accretion. Arguing for considerable non-axisymmetry, on the other hand, is the observation that almost all pulsars have high space velocities, which means that even the approximately half of all pulsars that must have been formed by the collapse of single (not binary) stars are somehow given a kick from the explosion. A particularly striking example[9] of this is PSR 1758-24, whose apparent velocity of some 1200 km/s is implausibly large to be explained by the breakup of a binary system. Theoretically speaking, it is not hard to get strong rotation: if the pre-collapse core has as much angular momentum as the Sun, and if the angular momentum is preserved in the collapse, it will have more than enough to excite instabilities. However, there may be mechanisms to remove angular momentum efficiently from a collapsing core.[10]

It is worth mentioning that the usual assumption that all gravitational collapses lead to supernovae that leave behind a pulsar or (rarely) a black hole may not in fact be justified. A rotation-dominated collapse may, by virtue of its slower time-scale and lower peak density, produce a weaker shock than a spherical collapse does. Since the best recent spherical supernova calculations[11] produce weak shocks that are only marginally able to propel the outer envelope away, a rotation-dominated collapse may in fact not expel the envelope. Instead of a spectacular supernova, there might be a rather weak display, and instead of a neutron star at the center there will in the end be a black hole. Therefore, neutron-star statistics may not give a good indication of what happens when black holes are produced. Even if an extreme Nakamura-Oohara scenario occurs only in 1% of supernovae, the $0.01 M_{\odot} c^2$ of radiation they produce could be detectable by second-stage interferometers at distances of up to 40 Mpc, a volume of space that contains a number of starburst galaxies and has many hundreds of supernovae per year.

The uncertainty over the strength of gravitational waves from supernovae may only be resolved by observations with large-scale interferometric detectors.

2.2 Coalescing Binaries

If two compact objects (neutron stars or black holes) find themselves in a sufficiently close binary system, they will spiral together under the action of gravitational radiation reaction until they merge. In the few second before merger, they are a powerful and predictable source of gravitational waves.[1] Since the gravitational radiation is predictable and carries considerable energy (about $5 \times 10^{-3}\ M_{\odot} c^2$ is emitted in the last two seconds of orbital motion), these must be regarded as the most reliable source that second-stage interferometers can be expected to detect.

Nevertheless, there are a number of uncertainties that we would like to clear up: the number of coalescence events per galaxy per unit time is very uncertain;[2] the gravitational radiation given off after the stars begin to coalesce is much harder to predict; and there is uncertainty about whether such events can be (indeed, possibly have already been) seen through the electromagnetic energy they emit.

- The *event rate* has to be calculated by extrapolating to other galaxies observations in our own of precursor systems: compact-object binaries that will coalesce within a Hubble time. There are now three such systems known: PSR 1913+16 (the original "binary pulsar"), PSR 2127+11C (in a globular cluster), and PSR 1534+12 (a field system that is almost a clone of 1913+16). By considering the selection effects that govern the detection of such pulsars, their binary lifetime, and current ideas about how they evolve, two groups[12, 13] have recently and independently reached similar estimates of the neutron-star—neutron-star coalescence rate. They conclude that there should at a minimum be 1 coalescence per 10^5 years per galaxy, which extrapolates to about 1 per year out to a distance of 100 Mpc. These groups also suggest on evolutionary grounds that the number of neutron-star—black-hole coalescences should be comparable. Given that a network of 4 second-stage interferometers could detect neutron-star—neutron-star coalescences out to beyond 600 Mpc (and neutron-star—black-hole coalescences two or three times further), it follows that they may see an event rate of something like one per day.

- When the stars get so close that Newtonian gravity is not adequate to describe their orbital motion, one can use *post-Newtonian corrections* to improve the pattern-matching and hence the detection rate. Calculations of this sort were first performed by Krolak[14], and recently they have been improved by Lincoln and Will.[15] These approximations will break down when mass transfer or tidal effects begin to be important, and after this it is a problem for numerical simulation.

- The first serious *simulations of the merger of two neutron stars* have been performed over the past couple of years by Nakamura and Oohara.[7, 8] These suggest that considerable radiation will be emitted, perhaps as much as $0.01 M_\odot c^2$. Because this occurs after the orbital wavetrain on which the detection of the system is based, this energy can be detected at relatively low signal-to-noise, and its characteristics will be very informative about the nature of the system. These calculations are preliminary, however, and can only be suggestive of the real answer, because they are done in the context of Newtonian gravity with gravitational radiation reaction, and cannot represent adequately the effects of the formation of a black hole horizon, for example. Moreover, it seems that some of the energy that the calculations predict is emitted by the system is radiated after it should have formed a black hole, which is unphysical. Nevertheless, they show that we should not be too surprised to see the coalescence radiation in many binary systems.

- When neutron stars merge, one ought to expect a considerable emission of electromagnetic radiation as well. This might come from the scattering of neutrinos to form electron-positron pairs, which subsequently decay to gamma rays. Just how much of this gamma radiation can get through whatever cloud of material is formed by the merger is difficult to calculate, but there have been some estimates.[16, 17] It is at least possible that coalescences could be a strong source of gamma rays, and could explain the puzzling gamma-ray burst seen regularly. Although the most popular explanation for these is that they result from some sort of accretion phenomenon on nearby neutron stars, the recently launched Gamma Ray Observatory will provide much more information. If the bursts turn out to be isotropically distributed, they will not be associated with local neutron stars (biased towards the Galactic plane), and may well then turn out to be coalescing binaries.

Coalescing binaries are likely to be so important for detectors that much work recently has been done on studying how to extract information from the signals and how to enhance the sensitivity of detectors to them.

- The theory of the *detection of coalescing binaries* is now being developed. Although the waveforms are predictable, they depend on two unknown parameters (the so-called mass parameter[2] and the phase of the arriving wavetrain), so one has to use a family of matched filters in order to detect them all. The uncertainties in the measurements of these parameters are not independent, and they also affect the accuracy with which one can determine the time-of-arrival of the signal. If there were no uncertainty in the parameters, the timing accuracy would be better than a millisecond[18] — but see Lobo[19] on the practical difficulty of achieving this. Errors in the parameters can degrade this. Recent calculations by Dhurandhar, Krolak, Schutz and Watkins[20] show that most of this degradation is in the absolute time of arrival of the waves at the Earth, and that the relative time of arrival of a signal at two different detectors can be determined almost as accurately as if there were no uncertainty in any parameters. This is important because the relative timing in a network of three or four detectors is the way that the position of the source on the sky will be measured. By measuring the position of nearby sources (closer than 100 Mpc) accurately, one can use coalescing binary observations to determine the Hubble constant with few systematic errors.[21]

- There has been interesting work recently on means of enhancing the performance of interferometers to detect coalescing binaries. These have grown out of Meers' idea[22] of "dual recycling". By adjusting the position of the signal-recycling mirror, one can tune the detector to a narrow bandwidth at any selected frequency. Meers, Lobo, and Krolak[23] have recently suggested that this tuning can be done dynamically, so that the interferometer can be tuned to the frequency of the radiation being emitted by a binary, and this tuning can be adjusted as the binary stars approach each other and orbit more quickly.

  This could lead to an improvement of a factor of 3 or more in the signal-to-noise of a source that is strong enough to be detected early, so that the dynamical tuning can be implemented. This might allow, for example, the determination of the Hubble constant with just one event, by improving the positional accuracy of the source on the sky so that it can unambiguously be associated with a group or cluster of galaxies. This is something that detector builders need to keep in mind: the technical demands do not appear to be very difficult, but the decision to implement such a scheme at the cost of losing broad-band sensitivity occasionally may be a difficult one. For more details, see Meers article in this volume.

2.3 Stochastic Background

In addition to a thermal relic gravitational wave background arising from the big bang in much the same way that the microwave background radiation arises, there are a number of other possible sources of a stochastic background of radiation. All of them require some sort of new physics for their generation, and for this reason a search for the stochastic background is one of the most interesting goals of interferometric detectors.

Radiation can be generated by phase transitions and by topological defects associated with gauge theories for fundamental physics. Topological defects include cosmic strings, which, if they seed galaxy formation, should also produce an observable background.[24] Cosmic texture[25] is another topological effect that has recently become interesting as a way of accounting for the observed large-scale structure of spacetime, and further work must be done to calculate the radiation that would be expected from it. A recent calculation of Turner and Wilczek[26] on bubble collisions suggested that this might be an additional source of observable radiation. Their calculation gives a relatively narrow band of radiation near 100–1000 Hz, with an energy density that should be easily detectable with second-stage interferometers.

Cross-correlation experiments between interferometers may be the only way to test many of the exotic theories of the early universe. Such experiments can make relatively broad spectral measurements at kiloHertz frequencies. A later generation of space-based detectors could probe the very interesting milliHertz region of the stochastic spectrum.

2.4 Pulsars and Wagoner Stars

Possible sources of single-frequency continuous radiation include slightly asymmetric pulsars and Wagoner stars[27], which are accreting neutron stars that have been spun up by accretion to the point where a non-axisymmetric gravitational-radiation-induced instability (CFS instability) sets in.

The onset of a gravitational-wave instability depends delicately on a number of factors. Any rotating star may be subject to such an instability if it has no viscosity, but for slowly rotating stars the instability grows very slowly.[28] Real stars do have viscosity, and this can also prevent the instability. Recent calculations by Lindblom and Ipser[29] (and by others — see references in that paper) have shown that the likely viscosity in neutron stars is sufficient to wipe out the instability except in young, hot stars. This means that the instability may still be effective in slowing down a newly formed pulsar if it is spinning too fast, but the instability may be hard to excite in an older star that is being spun up by accretion, unless the accretion itself leads to significant heating of the star. This work makes Wagoner stars less likely as sources of observable radiation.

The fact that newly formed neutron stars may indeed spin down this way makes it desirable to look for this spindown radiation after any gravitational collapse event. Finn and Shapiro[30] show that such radiation from a collapse in our Galaxy could be detectable by present prototypes. As mentioned in our discussion of supernovae above, there is no evidence that neutron stars are born rotating rapidly enough to be affected by this instability, but it is possible that a rapidly rotating neutron star is formed in a collapse event that eventually leads to a black hole, through the subsequent infall of material from the envelope. See also the calculations of Piran and Nakamura.[31]

Regarding pulsars, known pulsars are potential sources of gravitational waves, whose amplitude is limited by the fact that the waves must not carry away more energy than can be accounted for by the pulsar spindown. A recent survey of known pulsars[32] shows that, if second-stage interferometers can screen out thermal and seismic noise in order to observe at 10 Hz, there are many potentially observable pulsars.

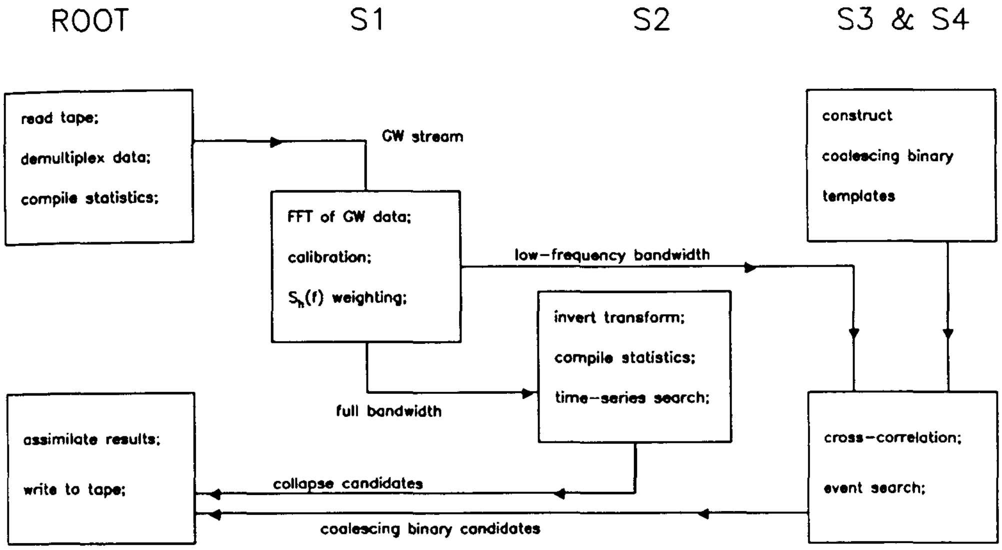
3 The Analysis of Data from the 100-Hour Coincidence Experiment

During the spring of 1989, the Glasgow and Garching interferometers were run in coincidence for a continuous period of 100 hours. Each detector operated at a noise level equivalent to about $h \sim 10^{-18}$. A full report on the results of the experiment is in preparation, but we can present here a preliminary view of some aspects of the analysis.

The Glasgow data were sampled at 20 kHz and recorded on about 30 high-capacity Exabyte tapes, multiplexed with various housekeeping streams, such as seismic and microphone monitoring data. The Garching data were sampled at 10 kHz and recorded on standard 9-track tapes, with little extra housekeeping data. We have since copied the Garching data onto Exabyte tapes at Cardiff.

One of us (Watkins) has written a prototype of an automatic data analysis program for the Glasgow data. This program reads data tapes and processes the data through several channels of analysis. The program runs on a network of five T-800 transputers hosted by a 386 PC. The transputers, which are full microprocessors with added communication links, allow parallel processing of the data through several stages. The division of tasks among the transputers, and the pipelining of data through the system, are described in Figure 1. The "root" transputer is the only one with communications to the outside world, so all input and output goes through it. It reads data in large blocks, de-multiplexes the different data streams (the Glasgow stream contain twice as much housekeeping data as signal-carrying data), and complies various indices of the performance of the housekeeping data. It passes the main data block to transputer S1, which performs a Fourier transform and uses calibration information in the data stream to remove frequency-dependent instrumental effects. This transputer then calculates the spectral noise density S_h of the signal stream at this time and weights the data's Fourier transform by dividing by S_h. Then it passes data

Figure 1: Flow diagram of the prototype data analysis program.

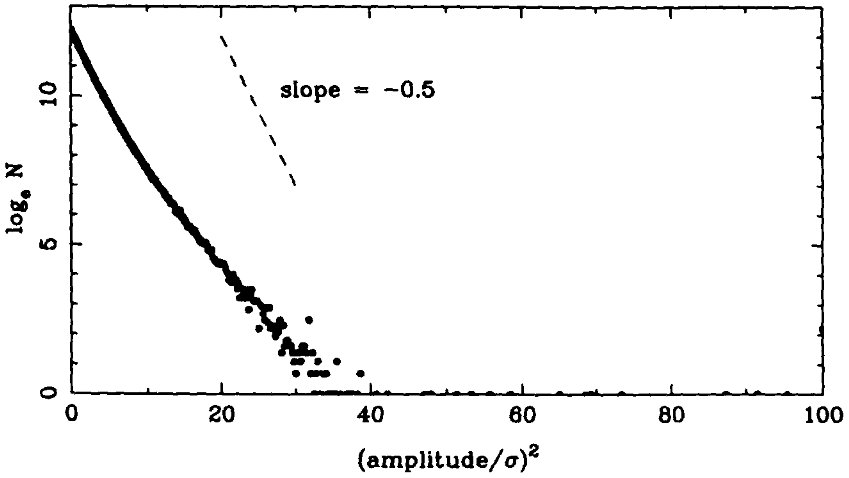


to the other three transputers. Transputer S2 receives the full-bandwidth weighted Fourier transform, takes its inverse transform, and searches this for "events", which are places where the time-series crosses some pre-determined threshold. It makes lists of events and accumulates statistics. Transputers S3 and S4 work in parallel: they both receive the lowest 1 kHz of the weighted Fourier transform and use it to perform a series of cross-correlations with various templates representing coalescing binary signals. These transputers then compile lists of events in each filter output. Because of the limited speed of the system, we are only able to apply about 16 different filters to the data. This is enough to give us experience with the handling of such data.

This pipeline runs at about 8 times real time: 100 hours of data can be analyzed in 800 hours of processing time, if the system works perfectly. In practice, the system has experienced severe delays due to the unreliability of our Exabyte drives, which reset themselves frequently and force the analysis to restart at the beginning of a tape. The entire data set has now been processed, however, and full lists of time-series and filter events are available on an Exabyte tape.

Another of us (Nicholson) has adapted Watkins' program to perform a similar analysis of the Garching data on a sequential machine (using one of our Sun Sparcstations). This program has also been completed, so that a full list of events in the Garching stream is available on Exabyte tape. The next step is to search event lists for

Figure 2: The amplitude distribution from the Glasgow interferometer



coincidences. This is being done right now.

A number of interesting features have emerged from this analysis. The first is that the noise in these interferometers does not seem to be a simple Gaussian distribution. Even when the detector is behaving optimally, the noise distribution seems to have a break at about 3σ, above which it seems to be roughly Gaussian but with a larger standard deviation. Figure 2 plots this distribution for a part of the Glasgow data. The axes are chosen so that a purely Gaussian distribution would show a straight line of slope $-1/2$. This is true of the upper part of the curve, but lower down the slope changes to $-2/7$, which corresponds to a Gaussian with standard deviation about 1.3 times larger. It is not yet clear why this arises, or even whether it is an instrumental effect. However, tests we have performed on simulated data seem to rule out the possibility that it is an artifact of our analysis method, with its recalibrations and weightings. But if it is true of the large-scale interferometers, then it will have a significant effect on their sensitivity, since thresholds for most observations will be above the break, and so will be governed by the larger standard deviation: thresholds will have to be set 1.3 times higher than if this effect were absent.

We have also observed that when the interferometer is not performing well, the non-Gaussian tail of the distribution gets much larger. This can in fact be a diagnostic for accepting or rejecting the data stream: the more nearly Gaussian it is, the better the detector is performing.

Finally, we have also found that a very large fraction of the noise events in the Garching stream are not explainable as simple random fluctuations, since they last as long as a millisecond. For data sampled at 10 kHz, one expects occasional isolated points above the threshold, but in fact here the data points consistently stay above threshold for 5 or 10 consecutive points. Again, the origin of this is not clear, but it will complicate the search for genuine coincidences between the two data streams.

We have learned a number of lessons from our analysis of the 100-hour data that should help with the design of systems for the full-scale interferometers. The first is that the requirement that one should analyze data in real time, when it is being collected at a rate of several tens of kHz, is very demanding on the input/output systems of computers. First, there is the problem of data recording. Standard 9-track tapes have no difficulty, but have a very low capacity for their physical volume. Optical discs have high capacity and random-access possibilities, but cannot yet write data at these speeds. High-capacity magnetic tape, such as the Exabyte tapes we use. offer rapid recording and large capacity in a small physical volume, but they are designed for archiving and computer backup, so they tend to perform erratically when used in a stop-start fashion for data analysis. This problem needs to be tackled before large-scale interferometers come on line.

A second lesson we learned is that multiplexing data on a fine scale (byte-by-byte) can considerably slow down the later analysis of the data. The transputer pipeline's throughput was limited by the time it took the root transputer to de-multiplex the data. It will be much preferable in the full-scale detectors to have data packaged up in larger chunks, which can be done if the recording system is equipped with suitable data buffers. This point has been amplified in a report written at Cardiff by one of us (Shuttleworth).[33]

A third lesson is that the data streams contain information that can be used as diagnostics for the performance of the detectors, and so they should be analyzed from this point of view as much as for their gravitational wave content.

As a result of tis experience, we are designing a new data format that will allow data to be stored and transmitted in large chunks flexibly. We are writing both data streams onto new tapes in this format, and we expect that after this is done further analysis of the data (such as for a full cross-correlation) will be easier and faster.

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