SYMPOSIUM 14: RADIATIVE SPACETIMES AND APPROXIMATION METHODS

B.F. Schutz
Department of Applied Mathematics and Astronomy,
University College Cardiff CF1 1XL, Great Britain.

Abstract. Talks presented at the symposium fell into three categories. Talks on the *quadrupole formula* strongly reinforced the consensus that the Einstein-Landau-Lifshitz-Chandrasekhar quadrupole approximation is robust and useful, and they clearly showed some new directions that research in the field is taking. Speakers about *gravitational radiation* widened the discussion to spacetimes with fully nonlinear radiation from fully relativistic systems. A closing section on *approximation methods* contained two papers on other aspects of weak field and/or slow motion spacetimes.

INTRODUCTION

Thirty-one abstracts were submitted for this session and twelve talks were presented. The field is clearly a healthy one. The controversies of previous GR meetings concerning the validity of the 'quadrupole formula' have largely been resolved, and research on that subject is taking new directions that promise a deeper understanding of radiating systems. I will summarize the three sections separately below.

It goes without saying that the division of subjects into different symposia is often of necessity arbitrary. Readers who want a good overview of the subject should refer also to the reports on Symposium 20, in which J. Anderson gave a review of work on the quadrupole formula and K.S. Thorne described an approach to equations of motion for compact bodies; on Symposium 11, where D. Hobill discussed the matching of numerical source calculations to analytic wave expressions; and to the plenary lectures of J. Ehlers and T. Piran.

THE QUADRUPOLE FORMULA

J.Winicour (MPI Astrophysik, Garching) The quadrupole formula

E.M.Kirk (U.Southampton, UK) An approach to a quadrupole formula error bound via Newtonian gravity on the null cone

T.Futamase (Washington U., St. Louis) The strong field point particle limit and the equations of motion for the binary pulsar system

J.L.Anderson (Stevens Inst. Tech., N.J.) Gravitational radiation damping of systems with compact sources

F.I.Cooperstock (U.Victoria, BC) Applications of the new forumula for gravitational radiation energy loss

G.Schäfer (Meudon) Gravitational radiation and generalized Lie-series

At GR10 it appeared that a consensus was emerging that the quadrupole approximation was a good one for nearly Newtonian systems, both for the near-zone radiation reaction and for the far-zone radiation amplitude. Work in the intervening three years has clearly strengthened that consensus. We have a new and very different null cone derivation of the far-zone formula (Winicour, Kirk), and we have new demonstrations that the formulae even extend to compact bodies like neutron stars and black holes, if they move in external nearly-Newtonian fields (Futamase, Anderson). The calculations here are still on the level of 'formal' methods, in the terminology of Ehlers' plenary lecture, although error estimates may soon be available (Kirk). Rigorous theorems in this subject still seem far away. Nevertheless, the quadrupole formula is impressively robust, not only because it is derivable in a variety of different ways, but also because it seems to give good results in some situations where it might be expected to fail.

The null-cone approach to the Newtonian limit described by Winicour is one of the most original ideas to have emerged in this subject recently. By adopting a null-hypersurface initial-value formulation before going to the limit $c \to \infty$, some connection between the Newtonian source and asymptotic null infinity seems to be maintained. The result is that a calculation of the Bondi news function in the limit $r \to \infty$ gives the standard quadrupole formula. Kirk, reporting work done jointly with J.A. Vickers, carries the Winicour formulation a step further. By using the Newman-Penrose equations, he obtains a simpler null-cone system, with the result that it seems possible to generate useful error bounds.

It is clear that these methods will stimulate much more work in the future. In this context, therefore, I think it will be important to pay attention to a subtle difference between the calculations of Winicour and Kirk. Winicour sets up his post-Newtonian expansion by taking the limit $\lambda \to 0$ ($\lambda \sim c^{-1}$) at fixed r, while Kirk allows for a limit at fixed λr. In the language of matching, Winicour's seems to be a near-zone limit, while Kirk can choose either the near or the far zone, depending on how he treats r. It is not surprising to see the quadrupole formula emerge from Kirk's far-zone limit, but how do we interpret it in the Winicour calculation?

It is my view that this is related to a point emphasized by Piran in his plenary lecture, that in his numerical calculations of gravitational collapse, he can see the gravitational wave amplitude taking shape in the near zone, well inside one gravitational wavelength. Piran's result applies to his particular gauge, but I have elsewhere (Schutz 1986a) shown that this happens in any gauge in the Newtonian limit. In this view, then the Winicour calculation gives us the most rigorous demonstration so far that by going well out in the near zone (Winicour's $r \to \infty$ limit) one can pick out the gravitational wave amplitude. As Piran remarks, the usual view is that the gravitational wave amplitude cannot reliably be identified except outside a few radiation wavelengths. Winicour provides a framework for revising this view, at least for nearly Newtonian systems.

Turning now to the motion of compact bodies, Futamase outlined an approximation method which shows that the motions of N bodies with arbitrarily strong internal gravity are the same as that of N weak-field nearly-Newtonian bodies, provided that the interior of each body is nearly stationary and the bodies are well separated. This is closely related to the work of Hartle & Thorne reported in Symposium 20, and it grows out of earlier work by D'Eath and Kates, but there are some differences. Where Hartle & Thorne solve the field equations in different regions and then match the solutions to each other, Futamase writes down a single implicit integral solution valid everywhere, which he then finds approximations to in various regions. Hartle & Thorne calculate the precession and deviations from geodesic motion caused by coupling of a body's multipole moments to external curvature; Futamase derives the Newtonian motion, the near-zone quadrupole radiation reaction, and the far-zone quadrupole radiation field. In all cases, the 'body-zone' ADM masses replace the Newtonian masses in the standard formulae. This shows clearly that the strong equivalence principle embraces 'gravitational potential energy' even in the generation of gravitational waves by bodies with strong internal fields.

Also closely related is the work reported by Anderson, in which he uses the EIH method to derive the result that compact bodies radiate energy and angular momentum by the 'quadrupole' formula if they interact weakly. The method uses integrals on surfaces around the bodies, for the evaluation of which it uses asymptotic expressions for the interior metric. In this respect it is only slightly less complete than the Futamase and Hartle & Thorne calculations. In compensation, however, the method is computationally efficient, gives finite integrals, and can be applied to black holes (which Futamase's method cannot be).

The talk by F.I. Cooperstock went some way toward resolving the differences between the general calculations of the quadrupole formula and his specific calculations of radiation from particular systems which were evident at GR10. He and R.H. Lim obtain the quadrupole formula in a particular limit and are investigating corrections in cases where the size of the source may not be small compared to the size of the "near zone".

G. Schäfer introduced a novel approach to the full problem of radiation generation in general relativity, using a canonical approach based on Lie series with retardations. This is an outgrowth of his earlier work on the ADM n-body problem.

In summing up this section, I would like to remark that the recent work is beginning to reveal new features of radiation in the Newtonian limit, going beyond the efforts at previous GR meetings simply to verify the quadrupole formula. Besides the work on wave formation in the near zone referred to above, there is the beginning of a systematic attempt to go beyond the quadrupole formula. If the next step may be called the 'octupole formula' - even though it contains more than just octupole radiation, such as the post-Newtonian corrections to the sources of the quadrupole radiation - then I believe its predictions may become testable by the end of the century. Observations of coalescing binaries with long-baseline interferometric gravitational radiation detectors promise an accuracy at the 1% level, which is where the post-Newtonian corrections to the orbit and to the radiation become important for these systems (Schutz 1986b). Achieving the octupole formula means, among other things, dealing with possible logarithmic terms in the post-Newtonian expansions, as discussed by Anderson in Symposium 20. Finally, as another direction for 'new' work, I should mention the work of Nahmad-Achar (1986) for which there was insufficient time at this meeting, in which he shows that angular momentum in gravitational radiation is free of supertranslation ambiguity in the Newtonian limit.

GRAVITATIONAL RADIATION

J.Bičák (Charles U., Prague) Radiative spacetimes with boost-rotation symmetry

J.L.Friedman (U.Wisconsin, Milwaukee) Gravitational radiation from rapidly rotating netron stars

E. Nahmad-Achar (U.Cambridge) Angular Momentum in General Relatity

A.M.Abrahams (U.Illinois) Mode-mode coupling in the implosion of Brill waves holes (which Futamase's method cannot be).

The talk, by F.I. Cooperstock went some way toward resolving the differences between the general calculations of the quadrupole formula and his specific calculations of radiation from particular systems which were evident at GR10. He and R.H. Lim obtain the quadrupole formula in a particular limit and are investigating corrections in cases where the size of the source may not be small compared to the size of the "near zone".

G. Schäfer introduced a novel approach to the full problem of radiation generation in general relativity, using a canonical approach based on Lie series with retardations. This is an outgrowth of his earlier work on the ADM n-body problem.

In summing up this section, I would like to remark that the recent work is beginning to reveal new features of radiation in the Newtonian limit, going beyond the efforts at previous GR meetings simply to verify the quadrupole formula. Besides the work on wave formation in the near zone referred to above, there is the beginning of a systematic attempt to go beyond the quadrupole formula. If the next step may be called the 'octupole formula' - even though it contains more than just octupole radiation, such as the post-Newtonian corrections to the sources of the quadrupole radiation - then I believe its predictions may become testable by the end of the century. Observations of coalescing binaries with long-baseline interferometric gravitational radiation detectors promise an accuracy at the 1% level, which is where the post-Newtonian corrections to the orbit and to the radiation become important for these systems (Schutz 1986b). Achieving the octupole formula means, among other things, dealing with possible logarithmic terms in the post-Newtonian expansions, as discussed by Anderson in Symposium 20. Finally, as another direction for 'new' work, I should mention the work of Nahmad-Achar (1986) for which there was insufficient time at this meeting, in which he shows that angular momentum in gravitational radiation is free of supertranslation ambiguity in the Newtonian limit.

GRAVITATIONAL RADIATION

J.Bičák (Charles U., Prague) Radiative spacetimes with boost-rotation symmetry

J.L.Friedman (U.Wisconsin, Milwaukee) Gravitational radiation from rapidly rotating netron stars

E. Nahmad-Achar (U.Cambridge) Angular Momentum in General Relatity

A.M.Abrahams (U.Illinois) Mode-mode coupling in the implosion of Brill waves

This group of talks spans a wide variety of investigations of radiation outside the post-Newtonian framework.

J. Bičak emphasized that the spacetimes with axial and boost symmetry are the only known exact solutions containing gravitational waves and sources. An infinite number of explicit solutions describing various "uniformly accelerated" multipoles is available at present, and they can be used for testing approximation methods and numerical codes. For example, an 'octupole formula' (with vanishing quadrupole terms) can be derived from exact solutions. There are always two incomplete generators of null infinity, and the ADM mass vanishes; but null infinity is topologically $S^2 \times R$, the radiative field peels appropriately, and these are liable to be the best analytic solutions with source and radiation that we will have for some time.

J.L. Friedman reminded us that rotating stars are subject to a gravitational-wave instability, and in particular that pulsars with millisecond periods may be spinning at the critical angular velocity for such an instability. This angular velocity depends on the neutron star equation of state, and he presented results for the critical angular velocity for a variety of equations of state and levels of viscosity, which tends to damp out the instability. The 1.6 millisecond pulsar (see the plenary talk by J. Taylor) is in the right range, although one example like this is not sufficient to establish that this effect is indeed the one that determines the angular velocity. A new pulsar with a 1.16 ms period may have been found recently, but the detection is not yet certain.

E. Nahmad-Achar presented new variational principles based upon angular momentum. After finding pseudotensorial measures of energy and angular momentum which can be evaluated in any co-ordinates (not just asymptotically rectangular ones), he examined what solutions of Einstein's equations extremize the angular momentum on a spacelike hypersurface or the angular momentum across a timelike hypersurface. In the latter case, a new method for calculating radiating axisymmetric gravitational collapse emerges: a metric plus perfect fluid is an axisymmetric solution of Einstein's equations if and only if it extremizes (under mild constraints) the angular momentum flux across the timelike hypersurface Φ = const, where $\partial/\partial\Phi$ is the axial Killing vector.

A.M. Abrahams showed how nonlinear effects in gravitational waves generate new waves. Reporting analytic and numerical calculations erformed with C.R. Evans, he showed that the pure $\ell = 2$ linear Brill wave Teukolsky wave) travelling in toward the origin emerges later with $\ell = 4,6,\ldots$ components. The numerical calculation was illustrated with a videotape of the results.

Related to this was the abstract submitted by C.D. Ciubotariu (Poly.Inst.Jassy, Romania). He discussed wave generation by wave-wave interaction in general relativity on arbitrary backgrounds, in the short-wavelength limit. This is closely related to the calculation of the Isaacson effective energy-momentum tensor, which may be regarded as the special case in which the two interacting waves generate a secondary wave of infinite wavelength. Unfortunately, Ciubotariu's abstract overstates the size of the wave-generation effect: it cannot significantly reduce the amplitude of the waves which are being looked for in gravity-wave experiments.

In summary, gravitational radiation research seems very healthy. The existence of variational principles that include radiating spacetimes may open up new analytic possibilities. Mixed numerical-analytic calculations may become more common (see also the report on Symposium 11). And Friedman's talk highlights an area of research which has new urgency: the calculation of linear normal modes of rotating relativistic stars. These are needed for two reasons: first as test-bed examples for the fully nonlinear, three-dimensional, fully relativistic hydrodynamical codes now under development; and second because developments in X-ray astronomy and in gravitational wave detectors may make the sorts of unstable stars Friedman discussed observable in the next decade.

APPROXIMATION METHODS

H.Pfister (U.Tübingen) Analysis of some characteristic physical effects in general relativity through perturbation of the Schwarzschild mass shell.

H.Okamura (Kogakuin U., Tokyo) Post-post-Newtonian approximation in general relativity

This group of talks uses approximations that do not involve radiation. Pfister discussed three approximate calculations, all of which perturb Schwarzschild mass shells. The first was 'gravitational shielding', in which the effect of external gravitational fields is reduced inside a spherical shell. The second was whether a rotating shell can be made to produce inside itself not only the Coriolis force of frame-dragging but also a centrufugal force of appropriate magnitude. The answer is that it can, and the demand uniquely determines the shell's shape. The third result was a calculation of the gravimagnetic repulsion of two nearby, even interpenetrating, rotating shells. It can balance their attraction if the shells are superimposed and relativistically rotating.

Okamura spoke on the derivation he and colleagues had given several years ago of the post-post-Newtonian equations of motion and Lagrangian for many-body systems. In particular he felt that recent work by Schäfer and T. Damour had taken insufficient account of their contributions. Schäfer responded to this during the discussion, saying that he and Damour had corrected errors and shortcomings in Okamura's work and had correctly referred to it (Damour and Schäfer 1985). A paper presenting the view of Okamura and colleagues will soon be published in Prog.Theor.Physics (Kyoto), written by T. Ohta and T. Kimura.

REFERENCES

Damour, T., and Schäfer, G. (1985), *Gen.Rel.Gravit.* 17, 879-905

Nahmad-Achar, E. (1986). Ph.D. thesis, University of Cambridge, Cambridge England

Schutz, B.F. (1986a) *In* Dynamical Spacetimes and Numerical Relativity, ed. J. Centrella. Cambridge: Cambridge University Press

Schutz, B.F. (1986b). *Nature* to be published.