SOURCES OF GRAVITATIONAL RADIATION

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ABSTRACT. There is, not surprisingly, considerable uncertainty in the theoretical predictions of the strength and rate of occurence of gravitational wave events of a potentially observable strength and frequency. Theory can at present only give an indication of what sources *might* be seen, together with limits on their strength and event rate. This review considers both the kind of information we can expect from observations of any particular class of sources, and the degree of confidence we have in the observability of that class. I begin with useful formulae for estimating the detectability of different sources; in a given detector, and provided optimal filtering can be used for sources with long wave trains, then the relative detectability of two sources at the same distance depends only on the total energy radiated and the dominant frequency of the radiation. I follow with a review of the most commonly discussed gravitational wave sources: gravitational collapse, coalescing binaries, pulsars, rapidly spinning accreting neutron stars, and the stochastic background. I include a discussion in detail of two subjects that have become interesting recently: the possibility that the rate of gravitational collapse events in our own Galaxy is high, perhaps one every one to three years; and estimates of bounds on the (highly uncertain) rate at which binary coalescences occur at relatively large distances. It is very probable that a network of four detectors built to today's design would detect at least a few events per year, and perhaps as many as several hundred thousand. The rate for systems containing at least one black hole may be significant too. I conclude with a discussion of the kinds of astrophysical information that observations of coalescing binary systems can provide: a value for Hubble's constant, information on the neutron-star equation of state, tests of general relativity, surveys of the mass distribution of the universe out to 0.5 Gpc or so, and (with further technical development of the detectors) perhaps even values for the deceleration parameter and the redshift at which star formation began.

1. INTRODUCTION

The development of designs for ever more sensitive broad-band gravitational wave detectors during the last five years has allowed theorists to re-

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examine the question of what sources future detectors would be likely to see; and the recent supernova in the Large Magellanic Cloud (SN1987a) has similarly reopened the question of how often relatively nearby events occur with a strength that would allow detection by the cryogenically cooled bar detectors that will soon be ready to do long-term observations. Because the subject of sources and detectors has been comprehensively reviewed very recently (Thorne 1987), I will give only a brief overview of the wide range of sources that theorists have studied, but I will include within it a more detailed discussion of the two points I have just referred to: nearby sources detectable by bars, and coalescing binary systems, which are theorists' current 'best bet' sources at large distances (greater than, say, 100 Mpc). But first I will address the 'detectability' of sources: what characteristics make one source easier to detect than another in a given detector?

2. DETECTABILITY OF SIGNALS

Given the inevitable uncertainties associated with predicting what gravitational wave detectors will see, it is useful to have simple formulae that allow one to estimate the likely strength of the gravitational waves that would come from a hypothetical source, and the detectability of those waves. In this section I give some formulae and use them to make some interesting comparisons among possible sources.

2.1. Characteristics of the source.

The fundamental measure of the amplitude of the gravitational wave is h, the tidal strain produced by the wave in the distance between free particles. A useful back-of-an-envelope formula for h produced by nearly-Newtonian systems is (Schutz 1984)

$$h \lesssim \phi_{int}\phi_N/c^4, \qquad (2.1)$$

where

$$\phi_N = GM/r$$

is the Newtonian potential of the source at the observer's distance (r), and ϕ_{int} is the typical value of the Newtonian potential *inside* the source. This expression provides an upper limit to the radiative field h, on the assumption that the motions giving rise to the radiation are driven by self-gravitational forces (ϕ_{int}). The upper limit is reached for systems with maximal asymmetry, but h will be well below the limit if the motions are nearly spherically symmetric.

The flux of energy in a gravitational wave is roughly

$$F \sim \frac{c^3}{16\pi G}|\dot{h}|^2 \sim \frac{\pi c^3}{4G}h^2 f^2, \qquad (2.2)$$

where f is the wave's frequency. For a simple wave of dominant frequency f and duration τ (and number of cycles $n = f\tau$), we find that the total radiance (energy per unit area) is

$$R \sim F\tau \sim \frac{\pi c^3}{4G} h^2 f n \sim \frac{\Delta E}{4\pi r^2} . \qquad (2.3)$$

2.2. Filtering wide-band data

If the effective bandwidth of the signal is smaller than that of the detector, then the signal to noise ratio (S/N) will be improved by optimal filtering (Davis 1988). The best filter is one which is as long as the signal. A useful estimate of the *best* amplitude signal-to-noise ratio is provided by the formula

$$\frac{S}{N} \sim \frac{hn^{½}}{\sigma} \sim \left(\frac{4R}{\pi f}\right)^{½} \frac{1}{\sigma} ,$$

where σ is the noise in a bandwidth equal to f, and n and R are as above. Using our expression for the radiance, we get

$$\frac{S}{N} \sim \frac{1}{\sigma\pi r} \left(\frac{\Delta E}{f}\right)^{½}. \qquad (2.4)$$

This signal to noise ratio is independent of n because we have assumed that we can integrate over all n cycles. This may, of course, be impractical for some sources, but our estimate still gives the best we can expect.

We can now compare the detectability of two different sources at the same distance, observed by different detectors:

$$\frac{(S/N)_1}{(S/N)_2} \sim \frac{\sigma_2}{\sigma_1} \left[\frac{\Delta E_1 f_2}{\Delta E_2 f_1}\right]^{½} . \qquad (2.5)$$

The frequency dependence of this formula may be strongly affected by the way that σ depends on f. For example, a simple interferometric detector (no reclycling) that is optimized to detect waves of frequency f has $\sigma \propto f^{3/2}$, while a recycling interferometer has $\sigma \propto f$ (Thorne 1987).

2.3. Examples

Consider first the *supernova* SN1987a. Suppose that a neutron star was formed with a large enough rotation rate to render it unstable to the gravitational wave instability (see Schutz 1987 and references therein). This would mean a spin period of the order of 1 ms. Then the star would be a source of 'spindown radiation' until it loses enough angular momentum to stabilize itself. Which would be easier to detect: the initial burst or the subsequent spindown radiation? Suppose, relatively conservatively, that the burst produced an energy $\Delta E_1 \sim 0.01\ M_\odot$ at $f \sim 1$ kHz. The spindown for the m=2 and m=3 instabilities takes place very rapidly, so observations begun a week or so after the event might see the m=4 instability. Supposing that the star spins down from a period of 1.3 ms to 1.4 ms in 3 months, it would radiate $0.001\ M_\odot$ at a frequency of, say, 1 kHz (very uncertain). If the two kinds of radiation are looked for with the same detector, then Eq.(2.5) tells us that

$$\frac{(S/N)_{\substack{\text{moderate}\\ \text{supernova}}}}{(S/N)_{\text{spindown}}} \sim 3. \tag{2.6}$$

Given the uncertainties about these estimates, it is clear that the spindown radiation may in fact be as detectable as the initial burst, especially if its frequency is lower than our estimate of 1 kHz. And if detectors had been looking for the spindown from the m=2 mode to m=3 in the first second or so after the burst, they might have seen it more easily than the burst itself. This is a possibility that should be borne in mind in future observations.

A second example is the *comparison of a supernova explosion and a coalescing binary at the same distance*, when observed by a recycling interferometer, in which the noise σ scales as f (Thorne 1987). If we take a supernova of $\Delta E \sim 0.01\ M_\odot$ at a frequency of 1 kHz, and for the coalescing binary system we use $f \sim 100$ Hz and $\Delta E \sim 0.01\ M_\odot$ (a typical value for the orbital energy radiated as the system is tracked from 100 Hz to 200 Hz), then we find that

$$\frac{(S/N)_{\text{supernova}}}{(S/N)_{\substack{\text{coalescing}\\ \text{binary}}}} \sim \frac{1}{30}. \tag{2.7}$$

This equation explains why coalescing binaries have replaced supernova explosions as the theorists' choice for the most likely source to be detected. Even if in a given volume of space the rate at which coalescences occur is as small as 10^{-3} of the rate of such supernovae, we may see far more coalescing binary systems because we can detect events in a volume of space 27,000 times larger.

3. A CATALOG OF SOURCES

In this section I will list and briefly describe the principal sources of gravitational radiation that are currently favored by theorists. Although considerable effort has been devoted to studying possible sources, it should be borne in mind that these lists are essentialy predictions, and the quality of our predictions is limited both by our imaginations and by the availablility of information on potential sources from astronomical observations in the *electromagnetic* spectrum. It would therefore be very

(2.1) and taking $\Phi_{int} \sim 0.1$ and $M \sim 1\ M_\odot$ for a neutron star, and $\Phi_{int} \sim 0.3$ and $M \sim 10\ M_\odot$ for a black hole. These lead to

$$h \sim 5 \times 10^{-22} \left(\frac{15\ \mathrm{Mpc}}{r} \right) \quad \text{for neutron stars (optimistic),}$$

$$h \sim 1 \times 10^{-20} \left(\frac{15\ \mathrm{Mpc}}{r} \right) \quad \text{for black holes (optimistic).}$$

It is hard to predict the waveforms from present calculations, but it may turn out here again that black-hole formation could produce a relatively standard waveform. If the collapse leads to the formation of a strongly nonaxisymmetric, tumbling core, then Ipser & Monagan (1984) suggest that the waveform could last as long as 30 cycles at about 1 kHz; however, their estimate of the amplitude of the waves is rather smaller than the optimistic one above, so that their predicted signal is just about as detectable as a burst at 1 kHz with the optimistic amplitude.

What are the chances of a collapse in our own Galaxy? If a collapse occurs at one fifth the distance to the recent supernova SN1987a, and if the present generation of bar detectors are on the air at the time, it will stand a good chance of being detected, particularly as one would probably also have a neutrino signal in coincidence. Until recently the accepted view has been that the collapse rate in our Galaxy should be similar to the pulsar birthrate or the supernova rate in nearby galaxies, *i.e.* about once per 30 years. But Blair (1988) argues, on the basis of a model for pulsar beaming that seems increasingly to be supported by observations, that the pulsar birthrate may be much higher, perhaps up to 1 every one to three years.

How can this be reconciled with the supernova rate? One possiblility is that not all collapses to neutron stars produce supernovae. (Because of the softness of the equation of state in the neutronisation regime, it is not possible to form a neutron star by quasistatic contraction of a white dwarf: neutron stars will always be formed by collapse.) Indeed, one might argue that there could be an *anti-correlation* between the emission of gravitational radiation and the production of a visible expanding cloud of gas. The maximum emission of gravitational radiation will come from rapidly rotating collapse, in which the collapsed star deforms into a tumbling ellipsoid or fissions into two or more smaller pieces temporarily. In such a situation, it may be harder to produce the rebound shock that drives off the envelope: with a larger ratio of surface area to volume such configurations may not trap neutrinos as effectively, and the more turbulent hydrodynamic processes may convert more of the released binding energy into heat rather than kinetic energy. Moreover, it may even be that the supernova rate *is* more like one every few years than one every thirty, but that most supernovae are not seen, either because the supernova luminosity function allows for a large number of less-luminous explosions, or because most supernovae occur in obscuring molecular clouds, or because supernovae are simply missed because they are transient events.

If collapses are really more common than has been assumed until recently, then -- as W. Fairbank has stressed -- the present generation of bar detectors has a real chance of making the first observations of surprising if gravitational wave observations did not turn up some completely unanticipated sources. The reader will find a more detailed discussion of many of these sources in Thorne (1987).

3.1. Gravitational collapse

Since gravitational collapse is the trigger for supernova explosions, the supernova rate gives at least an assurance that if we can 'see' as far as the Virgo cluster then there will be a few *potentially* observable events per year. There are two major uncertainties: (i) the degree of nonsphericity in a typical gravitational collapse, and (ii) the possiblility that the collapse rate is much higher than the supernova rate, with the majority of collapse events being electromagnetically 'quiet'. I will return to (ii) below. The problem of nonsphericity hinges in large part on rotation: if the collapsing core rotates moderately rapidly, the collapse will 'pancake' in an axisymmetric manner; if there is even more rotation, rapidly growing nonaxisymmetric instabilities may be excited, leading to a core that looks like a tumbling cigar or that even fissions into two or more pieces.

Numerical calculations of *rotating axisymmetric* gravitational collapse can now be done with reasonable accuracy, but the last word has by no means yet been heard. Müller (1982) finds that if the collapse forms a 1 $M_\odot$ neutron star, then typically only 10^{-7} $M_\odot$ are released in radiation, with a frequency of about 1 kHz. Piran & Stark (1986) show that if a 10 $M_\odot$ black hole forms, then one can expect perhaps 10^{-3} $M_\odot$ in gravitational radiation, at 1.5 kHz. Taking the collapse to be 15 Mpc away (the Virgo cluster) gives on these estimates

$$h \sim 5 \times 10^{-24} \left(\frac{15 \text{ Mpc}}{r} \right) \quad \text{for neutron stars, and}$$

$$h \sim 4 \times 10^{-22} \left(\frac{15 \text{ Mpc}}{r} \right) \quad \text{for black holes.}$$

On the other hand, if a collapse occurs in our galaxy, at a distance of 15 kpc, then the amplitudes are 10^3 times larger. This gets close to the sensitivity of present bar detectors. I will return below to the question of how likely a nearby gravitational collapse event might be. The waveform may be quite variable if a neutron star is formed, but Piran & Stark (1986) find that the waveform when a black hole forms is remarkably insensitive to the initial conditions, essentially because is is a superposition of the lowest few normal modes of the hole. This means that filtering might improve the detectablility of black-hole events by perhaps a factor of 2.

More promising for detection is a collapse which has enough rotation to go *nonaxisymmetric*. Here we do not yet have accurate numerical calculations to guide our estimates, although several groups are preparing attacks on this problem. We can get an optimistic estimate by using Eq.

if the binary were at 15 Mpc, it would be as detectable as a gravitational collapse that produced $h \sim 1.8 \times 10^{-20}$. As we concluded earlier, this means that we can see coalescing binary systems much further away.

The key question, then, is how often do coalescences occur relative to the supernova rate? Observations of pulsars in our Galaxy give us a clue, but one whose interpretation is still very uncertain. We know of one precursor of a coalescing binary system, namely the famous 'binary pulsar' PSR 1913+16. It consists of two neutron stars that will coalesce in less than 10^8 years. Although its eccentricity is large now, by the time it reaches an observable state it will have circularized. Given one such system out of about 400 known pulsars, the simplest estimate is that the coalescence rate is 1/400th of the supernova rate in any volume. This leads to a figure of about 3 per year out to a distance of 100 Mpc (Clark, *et al*, 1979). This is the rate that Tinto (1988) adopts in concluding that a network of four interferometers could observe 2000 coalescences per year. But this rate assumes that the binary pulsar is a typical pulsar, and that there is a steady state, with coalescences balancing births of coalescence precursors. I shall now discuss a number of uncertainties in this rate.

(i) *The binary pulsar is not typical.* There are now 7 known pulsars in binary systems, only one of which is a precursor to a coalescing binary system. But three of the five fastest pulsars are in binaries. Current thinking is that these are old neutron stars that have been spun up by accretion from the companion, and there are indications that they may have lifetimes up to 1000 times longer than most pulsars. This would depress the inferred coalescence rate by a factor of 100 to 1000.

(ii) *Small number statistics.* Since we see only one precursor system, there is some chance that we are lucky somehow, and that the real fraction of pulsars in precursor systems is 10 or even 100 times lower. With equal probability (on Poisson statistics) it could be 3 or 10 times higher. It is somewhat reassuring to note that there are 7 pulsars in binaries. Of these, PSR 1913+16 has the shortest orbital decay time, but two other systems (0655+64 and 1831-00) have likely gravitational-wave decay times that are of the order of a Hubble time. Our one precursor, therefore, may not be particularly unusual. I shall allow a ($1\ \sigma$) factor of 10 less or 3 more for small-number statistics. So far, therefore, we have a coalescence rate of between 3×10^{-4} and 0.1 per year out to 100 Mpc.

(iii) *There are selection effects in pulsar statistics.* These effects can go both ways. Since binary pulsars are preferentially short-period pulsars, and since people are looking hard for short-period pulsars (Lyne 1988), coalescence precursors may in fact be over-represented among all pulsars. However, there are severe observational selection effects against seeing short-period pulsars in binaries, because the Doppler shifting of the pulsar period smears out the sharp peak in the power spectrum that pulsar searchers look for (Lyne 1988). It may be fair to expect that binary coalescence precursors are actually more common than current observations would suggest. Quantifying this is, however, very difficult. I shall allow a factor of 1 to 10 for this, raising our rate to between 3×10^{-4} and 1 per year out to 100 Mpc.

(iv) *There may be no steady state, no equality between the coalescence rate and the precursor birth rate.* This is because we are dealing with systems that have lifetimes of the order of a Hubble time, so that the gravitational waves: they will be the only detectors capable of sustained operation for the next five to ten years. It must therefore be of high priority to establish a network of bar detectors committed to observations rather than to further technical development, detectors that will be on the air all the time. The establishment of GRAVNET (Blair, Frasca, & Pizzella 1988) is a most welcome development.

3.2. Coalescing binary systems

As should be clear from our discussion of detectability in §2, coalescing binary systems are very promising, essentially because they emit large amounts of energy from the decay of their orbits at a relatively low frequency over a long time. For example, two 1.4 $M_\odot$ neutron stars in circular orbits around each other will have an orbital frequency of 50 Hz -- and therefore a gravitational wave frequency of 100 Hz -- when their orbital radius is 150 km; the system still has a couple of seconds (!) to live before the radiation decay of the orbit causes the stars to collide or to tidally disrupt one another. The first discussion of these systems as sources of gravitational radiation was by Clark & Eardley (1977), but they only estimated the burst of radiation from the coalescence event. Our appreciation ofthe importance of the radiation from the orbit itself is due to Kip Thorne (see Thorne 1987). The signal we can expect (Thorne 1987; Krolak & Schutz 1988; Krolak 1988a,b) is conveniently expressed in terms of an r.m.s. average $\langle\rangle$ over the orientations of the detector and the source (but see Thorne 1987 for a better definition of a 'typical' signal strength). Consider a system consisting of two objects with total mass $M \times M_\odot$ and reduced mass $\mu \times M_\odot$, at a distance $r \times 100$ Mpc, whose Newtonian radiation comes off at the frequency $f \times 100$ Hz. One finds that

$$\langle h\rangle = 1.02 \times 10^{-23}\mu\, M^{2/3} f^{2/3} r^{-1} . \qquad (3.1)$$

Corrections due to possible magnetic interactions or tidal effects are negligible; for the post-Newtonian corrections, see Krolak (1988a,b). The rough estimate given by Eq. (2.1) is in fact very good here, since all the mass is moving nonspherically.

The timescale for decay of the orbit is given by

$$\tau = f/\dot{f} = 7.97\ \mu^{-1} M^{-2/3} f^{-8/3}\ \text{sec.} \qquad (3.2)$$

Because the orbital decay accelerates so quickly, the actual lifetime is 3/8 of this timescale. Notice that, since both h and τ are measureable, one can take their product to find r: the masses M and μ drop out. This ability to measure r is almost unique in astronomy, and it forms the basis of a new method of determining Hubble's constant (Schutz 1986a, 1986b). I will explore other consequences below. The long timescale means that the the number n of cycles in §2.2 is large: 0.6 $f\tau$, to be exact. So if a system consisting of two 1.4 $M_\odot$ stars is picked up when f = 100 Hz, its S/N can be improved by a factor of $n^{1/2} \sim 20$ over the S/N for a burst at 100 Hz. Compounding this with the fact that a recycling detector will be 10 times more sensitive at 100 Hz than at 1 kHz means that this signal will be 200 times more detectable than a 1 kHz burst with the same h; put another way, coalescence rate may depend as much on what systems were formed in the initial phase of star formation as it does on the present rates. Population II stars in spirals and ellipticals may well contribute to the coalescence rate, while they do not contribute to the (Type II) supernova rate. Since the average rate of star formation in our galaxy since its formation is probably some five times its present rate, and since including elliptical galaxies doubles the number of stars we have been considering, this effect would increase the coalescence rate by a factor of 10, provided that the distribution of coalescence times τ (number of systems in a given coalescence time interval) is flat, independent of τ. If it rises with τ, *e.g.* as τ^α for $\alpha>0$, then the correction factor is larger. Only if large numbers of systems with short coalescence times are formed would the present rate be dominated by recently-formed stars. The statistics for τ for the binaries in Table I are not very good, but they show no evidence for a preference for small τ. Accordingly, I will allow a factor of 10 to 50 for this correction, leading to a rate between 3×10^{-3} and 50 per year to 100 Mpc.

(v) *The gravitational collapse rate may far exceed the supernova rate.* Recent theories of the evolution of pulsar beaming have suggested that the rate of formation of pulsars has been underestimated, and it may be as high as 1 per three years in our Galaxy, compared to a supernova rate of 1 per 30 to 60 years (Blair 1988). This would raise the rate of coalescence by a factor of 10. Allowing for the possibility that this is not correct, we get a rate between 3×10^{-3} and 500 per year to 100 Mpc.

(vi) *There may be a significant number of black-hole coalescing binary systems.* We do not have enough information from pulsar statistics to estimate how many binaries contain a neutron star and a black hole of, say, 10-15 $M_\odot$ with a sufficiently short coalescence time. However, we can use the statistics of X-ray sources to give us a clue to how many there may be, because precursor systems probably pass through a stage in which they look like binary X-ray sources. Of the binary X-ray sources we can see, a few percent contain black holes of substantial mass (Hayakawa 1986). It may therefore be that in any volume of space, the rate of coalescence involving black holes may be a few percent of the rate for 'ordinary' two-neutron-star coalescence. From Eqs.(3.1) and (3.2) one can deduce that a system containing a 10 $M_\odot$ black hole will have a S/N some about twice as large as one with two neutron stars (Schutz 1986b), and so will be visible in a volume of space some 10 times larger. This may partly compensate the smaller event rate, so that these systems might increase the observed coalescence rate. Similarly, if two-black-hole systems coalesce at a few percent of the black-hole-neutron-star rate, they will be equally important: with an improvement of S/N of about 3 over the one-black-hole system, they will be visible in a volume of space 25 times as large. The observed event rate for black holes might be a substantial fraction of the two-neutron-star event rate. Of course, we may see none of these systems: it may be that companions of black holes never themselves evolve to black holes or even neutron stars. The effect of this on our estimated event rate is to raise the upper limit by a factor of 2, giving between 3×10^{-3} and 1000 per year for a system that can see two-neutron-star coalescences out to 100 Mpc.

Given all these uncertainties, Tinto's figure of 2000 events per year becomes something between 2 and 6 × 10^5 per year for a network of four detectors. It may be that only gravitational wave observations will show us where in this range the true rate lies. Nevertheless, the fact that by adopting the most pessimistic view on each point we still obtain a lower bound of a few per year is encouraging: there are very strong grounds for expecting networks of interferometers to detect coalescing binaries with great regularity.

3.3. Pulsars

Since pulsars have non-axisymmetric magnetic fields, they will inevitably be sources of gravitational radiation. But the amplitude of the gravitational waves produced by this mechanism is too small to be observable in the near future. However, we know little about the details of the structure of neutron stars, and in particular about the topography of their crusts. It is possible that some pulsars have lumps (slight ellipticity) of mass frozen in as the star cools and contracts. If the ellipticity is δ (the fractional distortion of the radius of the star) then h will be estimated by a variant on Eq.(2.1):

$$h \sim v_{rot}^2 \delta\, \Phi_{int} \Phi_N / c^6, \qquad (3.3)$$

where v_{rot} is the equatorial rotational velocity of the star. For a star of mass 1.4 $M_\odot$ and radius 10 km, emitting gravitational radiation at frequency f (twice the rotational frequency of the star), we have

$$h \sim 6 \times 10^{-22} \delta \left(\frac{f}{100\ \mathrm{Hz}}\right)^2 \frac{10\ \mathrm{kpc}}{r}\,. \qquad (3.4)$$

An upper limit on δ is set by the spindown rate of the pulsar: gravitational radiation cannot be carrying away more energy than the loss of rotational kinetic energy. For the *Crab pulsar*, this limit is $\delta < 10^{-3}$. Current observational limits by the Tokyo group (Owa *et al* 1986) are 3 × 10^{-22} on h and about 0.3 on δ. It is expected that current designs for LIGOs will (when resonant recylcing is implemented) reach perhaps $\delta \sim 10^{-6}$.

3.4 Wagoner radiation

The gravitational radiation instability mentioned in §2.3 can have a more spectacular effect than simply spinning a neutron star down rapidly. If the star is accreting from a binary companion, then it may acquire enough angular momentum to reach an instability point for a mode with a reasonably short growth time, say the m=4 mode. Further accretion will cause this mode to grow and to radiate angular momentum away, until a balance is reached between accreted and radiated angular momentum: the star then will remain a fixed-frequency 'beacon' on the accretion timescale. The intensity of this radiation will be proportional to the accretion rate, but so will be the intensity of X-rays from the source, so that the gravitational wave luminosity and the X-ray luminosity will be proportional. Wagoner (1984) showed that a source whose X-ray flux is F_x and whose gravitational radiation comes off with frequency f will have amplitude

$$h \sim 2 \times 10^{-27} \left(\frac{300\ \mathrm{Hz}}{f}\right)\left(\frac{F_x}{10^{-8}\mathrm{erg\ cm^{-2}\ s^{-1}}}\right)^{1/2}. \qquad (3.5)$$

Notice that this estimate does not contain the distance to the object: this is taken care of automatically in the decrease of F_x with distance. Since there are many sources of X-rays with fluxes above 10^{-10} erg/cm²s, and since LIGOs may well be capable of reaching $h \sim 10^{-28}$ with resonant recycling and long observing runs, it should be possible to search for these sources. However, the signal-to-noise ratio will be degraded unless the frequency f is known in advance. One way this might be observable is as a low-amplitude modulation of the X-ray flux. Present data do not show any modulations that would be candidates for this effect, but there is a proposal to fly an X-ray observatory that would have considerably more sensitivity for such effects than any previous satellites have had (Wood, *et al* 1986). If this satellite is built and discovers modulation in any source, that source will be a good candidate for the Wagoner radiation.

3.5 Stochastic background

The stochastic background of gravitational waves is any 'confusion-limited' wave field, where the time between receiving different and unrelated wave trains is less than the length of each wave train. Such a background must be sought by looking for correlations between independent detectors: it will appear as an irreducible, correlated 'noise' in the antenna. The detectors must be nearer than about half a wavelength of the radiation if they are to have optimum sensitivity. Michelson (1987) has studied this problem in detail. For bar detectors, correlations between detectors on the same site give the best sensitivity, provided seismic disturbances (also correlated) can be eliminated. For LIGOs, observing above 100 Hz, baselines between proposed European detectors would be ideal, but the proposed American detectors are rather far apart.

Possible sources of a stochastic background in the frequency range 10-1000 Hz include cosmic strings (Vachaspati & Vilenkin 1985; Brandenburger, *et al* 1986), primordial waves (reviewed by Thorne 1987), and binary coalescence and black hole formation in an early generation of stars ('Population III': see Bond & Carr 1984). At lower frequencies, close binary systems in our Galaxy and various phase transitions in the early universe may contribute.

It is usual to characterise the strength of the background at a frequency f by giving Ω_g, the energy density in the waves in a bandwidth $\Delta f = f$ as a fraction of the cosmological closure density. One of the firmest predictions of Ω_g is given by cosmic strings. Strings provide an attractive model for seeds of galaxy formation; with only one free parameter -- the mass per unit length μ of the strings -- they can explain a range of features of the observed masses and clustering properties of galaxies. Such seeds give off gravitational radiation, and for the best choice of μ one finds that $\Omega_g \sim 10^{-7}$ at observable frequencies. According to Michelson (1987), cryogenic bar-type detectors can reach this limit below 200 Hz, and

LIGOs may be able to get down as far as 10^{-12} at 50 Hz. It is clear that the testing of various theories for the stochastic background is one of the most interesting jobs for detectors being designed now.

4. ASTROPHYSICAL INFORMATION FROM COALESCING BINARIES

If coalescing binaries can be observed by LIGOs, we will learn more than just that gravitational waves exist. This is because, if the waves are observed with enough detectors, the waveform can tell us the absolute *distance* to the source. Since direct distance measurements are rare in astronomy, I will devote this section to mentioning some of the possibilities that this opens up.

4.1. The distance to a coalescing binary

The key to measuring the distance is in Eqs.(3.1) and (3.2), which give the mean $\langle h \rangle$ of the waves and the timescale τ for the change in their frequency. The masses of the stars enter these expressions in the same way, so that their product is

$$\langle h \rangle \tau = 8.13 \times 10^{-23} f^{-2} r^{-1} \text{ sec.} \qquad (4.1)$$

This is independent of the masses. Since the frequency is known, a measurement of $\langle h \rangle$ and τ determines the distance r to the source (given in this equation in units of 100 Mpc). Before we go on to see what we can learn from r, we need to address two points: First, can one measure $\langle h \rangle$, the r.m.s. orientation average of h, for any individual system? Second, how secure and reliable is the model of the binary system that leads to Eqs.(3.1) and (3.2)?

On the measurement of $\langle h \rangle$, it is clear that this is not directly measured. But if it is possible to measure the actual orientation of the source and its direction on the sky, then one can reconstruct $\langle h \rangle$ from this information. Now, a network of four detectors can determine this information (Schutz 1986a). It is not hard to see how the direction to the source can be inferred from time-delay information; the orientation of the source on the sky and the inclination of its orbital plane can be determined by measuring the polarisation of the wave.

On the reliability of the model, Krolak (1988a,b) has considered several possible complications to the basic model of a point-mass Newtonian binary in a circular orbit with quadrupole gravitational radiation reaction. By the time an orbit decays to the point where its period is a few milliseconds, it will be essentially perfectly circular: radiation reaction eliminates any initial eccentricity. Tidal effects between neutron stars have a negligible effect on the orbit until they begin actually to exchange mass. Magnetic effects are similarly negligible. Post-Newtonian corrections to the orbit can be significant, at around the 5-10% level, but these are not really a complication. In fact if they are large enough to influence the observations, then they can be solved for from the data, and this will give independent information that determines the individual masses of the stars. It seems, therefore, that the model is exceedingly robust and that the distances inferred from it are likely to be essentially free from systematic error.

4.2 What we may be able to infer from observations

As Tinto (1988) shows, a network of detectors may be able to see a sizeable fraction of all coalescing binary events out to 1.5 Gpc or even further. The following discussion of the implications of this is based upon Krolak & Schutz (1988).

(i) *Hubble's constant.* If the event rate is high enough to give several events per year from within 100 Mpc, then Schutz (1986a) has shown that an accurate determination of Hubble's constant is possible. This is because such events have a signal-to-noise ratio sufficiently large that a determination of the distance to the system accurate to a few percent is possible, and of the direction to $\pm 3^\circ$. If a search of this window reveals an optically visible supernova-like event from the actual coalescence of the two stars, then by measuring the redshift of the 'host' galaxy one can determine H_0 to a few percent, much better than we know it today. In the absence of any optical counterparts, a statistical method will suffice in the long run (after 10-20 events are recorded).

(ii) *Standard mileposts.* If an optical identification can be made as described above, the determination of the distance to the host galaxy provides a milepost for the calibration of a variety of distance measures and intrinsic luminosity measurements.

(iii) *Masses of neutron stars and black holes.* It is clear that, unless we detect the post-Newtonian corrections to the radiation, we can measure only the combination $\mu\ M^{2/3}$, which I shall call the mass parameter of the system. Provided that the event rate is high enough to generate good statistics on the mass parameter (a few hundred events per year would probably suffice), one may be able to infer the maximum mass of neutron stars. The existence of massive black holes in binaries would be clear if we observe large values of the mass parameter. If the post-Newtonian terms can be detected, even more direct information will emerge.

(iv) *Mass distribution in "local" region of the universe.* Events out to, say, 500 Mpc (z about 0.1 - 0.2) can be used to test for isotropy, homogeneity, superclustering, and the existence of voids. Binary coalescences ought to be randomly distributed among stars (whose distribution may or may not coincide with that of bright galaxies), so they should be ideal tracers of the stellar population.

(v) *Tests of general relativity.* The information from an observation by four detectors will test Einstein's predictions regarding gravitational wave polarization: four detectors overdetermine the solution for the wave, so any inconsistency among them would be evidence for polarization states not allowed by general relativity (Schutz 1986a). If an event is accompanied by the kind of optical counterpart discussed above, then a measurement of the speed of propagation of gravitational waves is possible: since the optical emission should brighten up within a day after the gravitational waves leave, the arrival at Earth of the two emissions with a day or so separating them would be evidence that the gravitational waves travel at the speed of light to an accuracy of better than one part in 10^9. Coalescence events provide other tests as well: the post-Newtonian terms predicted in Eqs. (1) and (2) might be independently detectable in radiation at *four* different frequencies, and must be consistent with one another. A two-black-hole coalescence event would be the strongest evidence one could imagine for the existence of black holes: not only would it identify the holes, but it would also test the predictions of general relativity regarding their radiation in the strong-field limit. (By the time observations are possible, numerical calculations should have determined these predictions to a high accuracy: Miyama *et al* 1986.)

(vi) *Neutron-star equation of state.* By following a coalescing binary system until the S/N drops to about 1, one may be able to get information about the dynamics of the actual coalescence of the two objects: when mass transfer begins, whether the coalescence is accompanied by collapse to a black hole, etc. Coupling this with numerical simulations should constrain the neutron-star equation of state. Bursts of gamma rays may also be detectable in coincidence with coalescence events (Blinnikov, *et al*, 1984). At the great distances involved, this could improve the above test of the speed of the waves by a factor of 10 or more in accuracy.

If LIGOs can improve their sensitivity by using technology that develops over the next ten years, then observations can be pushed to larger distances. Let us suppose (rather arbitrarily) that an improvement in sensitivity of a factor of 10 is possible, perhaps by a combination of higher laser power, better mirrors, longer baselines, the use of squeezed light (Leuchs 1988), and active seismic isolation. Neutron-star binaries can then be seen to a luminosity distance of 8 Gpc, which corresponds to a redshift larger than 1. The event rate will not be a problem with such a volume of galaxies: even on the most pessimistic assumptions, there would be thousands of events per year. Black-hole binaries would be seen at essentially any redshift, and those at redshifts of order unity would have S/N > 20. Such observations then open up a new set of possibilities: the measurement of the deceleration parameter and the mass density of the universe, the detection of differences in the formation rates of neutron stars due to chemical evolution, gravitational lensing of coalescing binary events, and the observation of the redshift at which star formation begins. See Krolak & Schutz (1987) for details.

These kinds of observations make a strong case for continuing the effort to develop LIGOs, even if that effort requires ten or more years of work on large systems to get them to their ultimate potential. If bar detectors are the observing systems for the near future, LIGOs seem to be the gravitational wave observatories of the future. At present, general relativity is important to astrophysics as an interpreter of observations made primarily in the electromagnetic spectrum. When observations like these begin to be made, general relativity will enter a new epoch as a provider of new and qualitatively different observational data to astronomy.

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