SOURCES OF GRAVITATIONAL RADIATION FOR DETECTORS OF THE 21ST CENTURY

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Abstract

This article reviews recent developments in the theory of sources of gravitational radiation for detectors that are now expected to operate after the year 2000. This includes new evidence that supernovae are non-spherical; binary evolution calculations that suggest that neutron-star binaries are more common; and low-frequency sources that could be seen by the recently-proposed LISA space-based detector.

1. Introduction

It is a great pleasure to speak at a meeting convened to honour Eduardo Amaldi. His enthusiasm for fundamental research of all kinds, and in particular for gravitational radiation, inspired many of us to work hard to make the field a reality. The leading role played today by the Rome/Frascati group in developing sensitive detectors is the best tribute to his memory that any scientist could wish for.

At this meeting there will be many talks showing the remarkable strides that have been made in the development of detectors of all kinds: bars and their new offspring, spheres; interferometers; and the new space proposals. I shall address in this talk some of the recent theoretical developments that affect our current predictions of what these detectors could actually see.

As a theorist, I am not so tightly constrained by the caution that experimenters exercise when they try to anticipate the likely performance of their detectors! Rather, I think that part of the role of the theorist is to provide realistic but challenging goals for detector development. So in this talk I will look ahead toward the possibility of detecting a large range of sources, with different kinds of detectors. Rather than review the whole area,[1] I plan to update developments that seem to me to be particularly significant.

2. Supernovae and Gravitational Collapse

As the most violent events known to astronomy, supernovae were the driving motivation for the early development of bar detectors. They are still the major target of bar detector development, and are of course an important source for ground-based interferometers as well. It is therefore ironic and frustrating that, 30 years after Joe Weber built his first bar detector,[2] we can say little more with certainty about this source than we could then.

The problem is that spherical motions do not emit gravitational waves,[3] and modern computers are still not able to perform realistic simulations of gravitational collapse in 3D, including all the important nuclear reactions and neutrino- and photon-transport. Spherical collapse codes do reasonably well on predicting the appearance of the expanding shell of material ejected by the supernova, but they tell us little about the dynamics of the collapsed core, where the gravitational waves would be generated.

If there is significant nonsphericity, it would presumably be generated by rotation. Instabilities can turn rapidly rotating, axisymmetric collapsing cores into tumbling triaxial (American or rugby) football shapes, which could emit powerful gravitational radiation. Therefore, many attempts to resolve this uncertainty look for evidence that there was considerable rotation in a supernova core collapse. What is the current thinking on this problem?

2.1. Pulsar Velocity Dispersion

Pulsars are presumably formed in supernova explosions. If the collapse is asymmetric, then perhaps this shows itself in properties of pulsars. Lyne and Lorimer[4] have looked carefully at data available for the spatial velocity of pulsars, and they have found that the mean velocity dispersion of pulsars is much higher than previously assumed. They have raised the mean spatial velocity by a factor of 3 to 450 km/s.

This figure is higher than the escape velocity from the galactic disc, and probably from the whole galaxy (depending on the amount of dark matter). Previous estimates were too low because they neglected the selection effect that the observed pulsar population includes a disproportionately large number of old pulsars that had relatively small initial velocities and that therefore stayed close to the galactic plane. With improved observations of pulsar velocities, Lyne & Lorimer were able to correct for the selection effect and obtain a more accurate result.

This remarkably high value for the pulsar velocity dispersion has many ramifications. For example, gamma-ray burst models: if pulsars normally escape from the Galaxy after they are formed, then old pulsars form a halo around the Galaxy, and they may therefore be the sites of gamma-ray bursts and still be as isotropic as the CGRO observations indicate.

In the context of gravitational waves, we must ask how pulsars acquired such large velocities. The normal explanation for their spatial velocities is that the supernova explosion that formed them broke up a binary system, leaving the pulsar with roughly the velocity of its progenitor star in its binary orbit. Since most stars are in binaries, this is plausible as long as the pulsar space velocities are comparable to orbital velocities in close binaries. But 450 km/s is comparable to the orbital speed of a planet orbiting the Sun at its surface, and binaries consisting of a main-sequence star and a giant that is about to become a supernova would have to be less compact than that.

The inevitable inference must be that the neutron star receives a "kick" when it is formed. This kick must be of the order of several hundred km/s, and it must come from some asymmetry in the dynamics of the supernova explosion. This is strong evidence that supernova explosions have considerable asymmetry. However, it is not conclusive evidence.

It is likely that such a kick comes from the asymmetric emission of neutrinos. This is plausible, since the collapsed core is only moderately "optically" thick to neutrinos, so a hydrodynamic asymmetry in the collapse could make certain parts of the star more transparent to neutrinos than other parts. Then there are two minimal statements that one can make about a supernova explosion that gives a neutron star a kick of, say, 300 km/s.

First, if the asymmetry were in the form of a perfectly collimated beam of neutrinos, then the star would acquire a speed of $v = 300$ km/s if the neutrinos carry a fraction v/c of the neutron star mass-energy. This works out to about 0.15% of a solar mass converted into neutrino energy.

Second, if this process happened on the 'bounce" timescale of about 1 ms, then there is a *minimum* gravitational wave amplitude of about 1×10^{-21} for a supernova at a distance of 1 kpc. The argument is simple:

$$h \sim \frac{\ddot{I}}{r} \sim \frac{1}{r}\frac{d^2}{dt^2}\int \rho\, x\, x\, d^3x \sim \frac{1}{r}\frac{d}{dt}\int \rho\, v\, x\, d^3x, \tag{1}$$

where I have omitted indices and factors of order unity. The last expression involves the time-derivative of the first moment of the momentum in the star. If the momentum separation between the streaming neutrinos and the bulk motion of the star occurs in a time Δt, then the value of h will be

$$h \sim \frac{MvR}{r\Delta t}. \tag{2}$$

Taking Mv to be the momentum of a neutron star travelling at 300 km/s, R to be the neutron-star radius, and Δt to be 1 ms, we arrive at the above estimate of the gravitational wave amplitude.

Neither of these conclusions is particularly exciting, but they represent only lower bounds that assume that the asymmetries are the smallest possible to fit the observations. A realistic collapse will not emit just one jet of neutrinos: there will probably be several directions of strong emission, with the net balance of neutrino momenta leading to the residual kick of the star. All of these asymmetries will radiate gravitational waves. The real question is whether the gravitational radiation produced by these asymmetries emerges on the collapse's hydrodynamic timescale of 1 ms or on the neutrino diffusion timescale of about 1 s. I would incline toward the former, because I find it hard to believe that the collapsed core could maintain a significantly non-ellipsoidal shape for as long as 1 s. If this is true, then one might expect gravitational wave amplitudes considerably larger than the lower limit arrived at in the previous paragraph.

Another possible mechanism for creating the neutron-star momentum might be direct anisotropic emission of gravitational waves. This leads, of course, to much higher estimates of amplitudes, or at least of detectability. If the asymmetry in gravitational waves represents 10% of the total gravitational wave emission, then the waves carry away about 1% of a solar mass in energy. If this energy comes out in a burst on the hydrodynamic timescale, then the amplitude would be around 10^{21} at the distance of the Virgo Cluster. And if the asymmetry builds up over a longer timescale, it will be just as easy to detect using matched filtering as the burst radiation would be.[5]

All of this just points to the need for better numerical simulations of gravitational collapse with rotation and neutrino transport. Without such calculations, we are restricted to back-of-the-envelope estimates, which are tantalising but not satisfying.

2.2. A Fast Optical Pulsar in SN1987A?

The supernova that occurred in the Large Magellanic Cloud in 1987 is a test of many aspects of our supernova modelling. The neutrinos from it confirmed the basic picture that a neutron star forms during the collapse. A number of observations, including the most recent ones showing rings near the supernova, suggest that it was very non-symmetrical. But there is so far no evidence of a radio or X-ray pulsar in the remnant. This is somewhat puzzling: even if the direct pulsed emission were obscured by the remnant or were beamed away from our observing position, we would expect to see the extra energy being put into the remnant by the pulsar, but there is no evidence for it.

However, unpublished data[6] suggest that there may in fact be a 467 Hz *optical* pulsar in SN1987A. The data come from observations with 3 different telescopes, including the Hubble Space Telescope, and the three are reasonably consistent. The data show a weak and intermittent periodicity in the light from the supernova remnant, and in fact indicate a rather complex pattern of modulation. If the pulsar is real, then we can conclude that means rotation must have had a strong effect in this supernova. This is in any case a peculiar supernova, and if it has a fast pulsar may be one indication that there is a subpopulation of supernovae that are strong sources of gravitational waves. However, as long as the data are unpublished, and unconfirmed by other groups, one must treat this with caution.

3. Coalescing Binaries

Coalescences of compact binaries consisting of either neutron stars or black holes are one of the most important potential sources for ground-based interferometers. They may also be detectable in the future by the new spherical solid antennas that will be discussed at this meeting. What makes them detectable is that we can model reasonably well the waveforms expected from the inspiral phase, as the two stars get closer and closer because of the emission of gravitational waves by their orbital motion.

We would also like to detect radiation from the "plunge" phase — after the stars reach the last stable orbit and then fall rapidly towards one another — and from the merger event, but both of these phases are not yet well-understood theoretically. Moreover, neither of these final phases will be likely to generate more signal than the inspiral phase, so the detectability of such systems rests on tracking their orbital emissions. Efforts to build interferometers with good low-frequency sensitivity down to, perhaps, 10 Hz will be particularly helpful in detecting coalescing binaries, since they emit much more of their power at low frequencies. A typical neutron-star binary can be tracked for several minutes if we pick up its signal at a few tens of Hz. Constructing good theoretical waveform templates for such an event will not be easy, because post-Newtonian effects on the orbit will be important. But it now seems that this can be done to sufficient accuracy to allow events to be detected with nearly the theoretical maximum signal-to-noise ratio.

The major uncertainty is the event rate: how many of these rare events should we expect? Our estimates of event rates are based on careful studies of pulsar detections in binary systems.[7, 8] Such systems are hard to detect, both because the Doppler effect of the orbital motion washes out the periodic pulsar signal, and because in any case pulsar surveys cover only a small portion of the Galaxy.

The Hulse-Taylor pulsar, PSR1913+16, is the prototype of a binary coalescence precursor. There are in fact now three known such binary pulsar systems with orbital decay times less than the age of the Universe. Their statistics suggest a rate of one neutron-star binary coalescence per 10^5 yr in the Galaxy. By extrapolation to other galaxies, this tells us that we have to be able to detect systems out to about 100 Mpc to get one event per year. This will be possible with stage-2 LIGO and VIRGO interferometers, but not at stage 1.

This rate is very much a lower limit, since it is based on pulsar detections, which sample coalescence precursors some 10^8 yr before they actually coalesce. Recently, two independent theoretical studies of binary evolution suggest[9, 10] that there may be a population of neutron-star binaries that are formed with much closer separations, so that they decay through gravitational radiation on a much shorter timescale. The decay time would, in fact, be shorter than the expected birthrate in the Galaxy, so that at any one time one would not expect to see such a system. Nevertheless, the

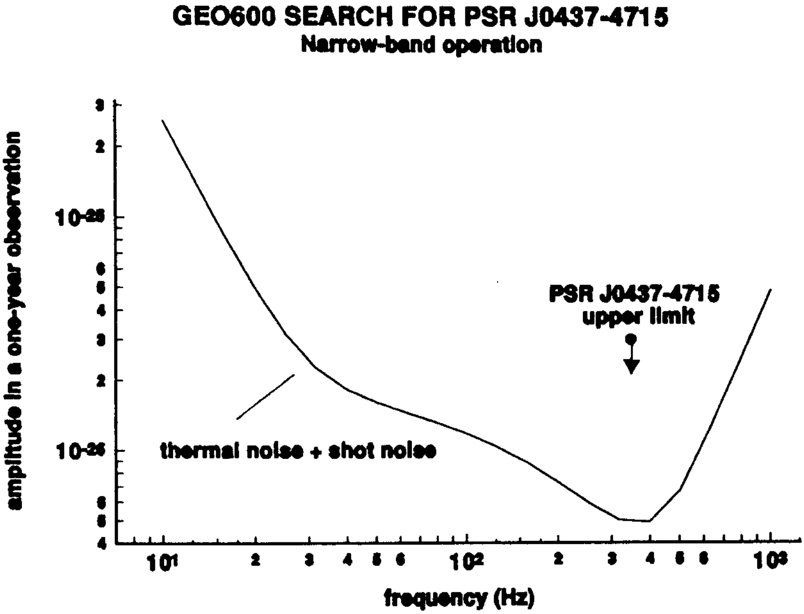

Figure 1: GEO600 sensitivity for pulsars.

birthrate is suggested to be so high that, when integrated over a Hubble time, such systems contribute an event rate 10 to 100 times as large as for Hulse-Taylor-type systems.

If this stands up to examination, then it will bring the nearest coalescence in one year in to the Virgo cluster, within range of the first-stage detectors, including GEO600, which Karsten Danzmann will talk about in detail at this meeting. This would completely revolutionise our expectations about the science we could do with these systems. Stage-2 interferometers would have enormous statistics, and would not only be able to determine the Hubble constant[11] but might also have enough statistics to see the effects of the deceleration parameter.

4. Pulsars

Pulsars will be strong radiators if they can sustain nonaxisymmetries. Conventional mountains on the solid crust may not be big enough, but large-scale irregularities frozen in at birth may still exist. Of course, old neutron stars may also be radiators, provided they have not already spun down, and provided their high spatial velocity (above) does not remove them from the Galaxy. Searching for continuous-wave sources may be the only way to find old neutron stars.

How strong would a pulsar signal be? In the absence of strong theoretical constraints, we can only use the spin-down rate to set upper limits for known pulsars. It seems well-established that the spindown is dominated by magnetic braking or the emission of relativistic particles, but there is no reason to think that gravitational waves could not be comparable, say a factor of 2 less energy.

On these grounds, two new pulsars at high frequencies may be excellent candidates for detection. The recently-discovered pulsar PSR J0437-4715[12] is the second-nearest known pulsar to the Earth, only about 150 pc away. Although it is not slowing down quickly, its proximity makes its spindown limit lie well above the expected sensitivity curve of GEO600, or the similar ones for LIGO and VIRGO. The situation for GEO600 is shown in Figure 1. Of course, the gravitational radiation may be well below this upper limit, but this is the first pulsar which we are likely to be able to set interesting limits on.

The possible SN1987A pulsar mentioned above has a much larger rate of spindown, but it is far away, so its upper limit lies below the sensitivity that GEO600 can achieve with broad-band power recycling, which will be its normal mode of operation. But the limit still lies above the thermal expected noise at that frequency, so with narrow-banding (signal recycling) GEO600 would be able to reach the limit. If the observations on this pulsar are still regarded as plausible by the time GEO600 operates, an effort will be made to implement signal recycling and look for it. None of the larger detectors would be likely to be in a position to do this for many years after GEO600, since signal recycling does not figure in their stage-1 plans.

5. Stochastic Background: An Ideal Bar-Interferometer Experiment

Astone, Lobo, and I have recently done a study[13] of the different observing possibilities for bars and interferometers working together. Remarkably, despite the coexistence of these two types of instrument for 15 years, no-one seems to have looked systematically at this question before. I will mention here only the conclusion that I feel is the most interesting and exciting: that it could well be worthwhile doing a joint cross-correlation experiment to search for stochastic radiation.

A cosmological background of gravitational radiation is one of the most interesting things that our detectors can look for: it is the earliest glimpse we will ever have of the Big Bang. If it is strong enough to detect with planned instruments, it will probably carry information about cosmic strings or other processes that acted as seeds for galaxy formation.

Searches for this background using two interferometers are a central part of the planning for LIGO and VIRGO, and indeed in Cardiff we are currently performing a search in the data from the Glasgow and Garching prototypes that was taken in 1989. Since the background radiation is stochastic, it is just a noise in the detectors, and the only way to be sure it is not instrumental noise is to show that it is correlated between two detectors. One cross-correlates data over a wide bandwidth and tests whether the correlated noise is significantly different at zero time-delay than at other time-delays. A significant excess noise at zero delay would be evidence for a background, provided the detectors were far enough apart not to be subject to other, terrestrial sources of correlated noise.

If the two detectors are an interferometer and a bar, the correlation can only be done over the bar's bandwidth, so the data from the interferometer will have to be filtered to this bandwidth before the correlation is performed. What we have found, however, is that the resulting sensitivity depends only on the broad-band burst sensitivities of the two detectors, and *not* on the bar's bandwidth. In other words, a 10^{-21} bar and a 10^{-21} interferometer can do just as good a search as two 10^{-21} interferometers. Here is how the calculation goes.

Any detector can be described, to a first approximation, by giving its bandwidth B and its spectral noise density S_h in that bandwidth. By S_h I mean the power spectral density in the detector output if it is normalised to the gravitational wave amplitude h. This is the only sensible way to compare detectors that have very different technology. Now, we normally characterise a detector, to the same first approximation, by its *broadband burst sensitivity*, which is the amplitude h_b of a broadband "supernova" burst signal that produces a 1-*sigma* response in the detector. The burst is assumed to be an unstructured signal of duration τ_b. Therefore it has a flat spectrum across a bandwidth $1/\tau_b$, which is usually taken to be about 1 kHz.

Now, the interferometers that are currently planned, at least at stage 1, will have effective bandwidths roughly equal to $1/\tau_b$. Such an interferometer will therefore receive a mean-square gravitational wave signal of h_b^2 in this bandwidth. This has to compete with the noise power of S_{int}/τ_b, where S_{int} is the power-spectral-noise density of the interferometer. The signal is just detectable at the 1-σ level if S_{int} is given

$$S_{int} = h_b^2 \tau_b. \tag{3}$$

A bar that has a much narrower bandwidth $B_b \ll 1/\tau_b$ receives a mean-square gravitational wave signal equal only to $h_b^2(B_b\tau_b)^2$ in this bandwidth, where its noise power is $S_b B_b$. Equating these gives a spectral noise density of

$$S_b = h_b^2 \tau_b^2 B_b = (\tau_b B_b) S_{int}. \tag{4}$$

This is not the usual way that bar detectors are described, but it is the most useful for our purposes.

Now, the cross-correlation of two detectors (labelled 1 and 2) produces a signal-to-noise ratio that depends on the bandwidth B and duration T of the observation, as well as the gravitational wave background power spectral density S_{gw} (remember that this background is just another source of noise in an individual detector), and of course the noise in each detector, S_1 and S_2. In addition, there are geometrical factors: one for the relative orientation of the two detectors, and another for the separation of the two detectors. The separation matters because waves that arrive at one detector significantly before the other will not be correlated in their zero-delay signal. (They will produce a correlation at the appropriate time delay, but this will be masked by waves coming from other directions, that are not correlated at that delay.) If detectors are separated by more than a reduced wavelength of the radiation, the solid angle from which waves may come and still be correlated is significantly reduced, so the correlation falls off.

If we neglect the geometrical factors, then the cross-correlation produces a signal-to-noise ratio of roughly

$$\frac{S}{N} = \left[\frac{S_{gw}^2}{S_1 S_2} BT\right]^{1/4} . \tag{5}$$

This sensitivity increases with the bandwidth, not surprisingly. This has led many people to assume that interferometers should be better than bars for such experiments.

However, if we now apply this to the bar and interferometer described above, and substitute for their spectral noise densities the expressions in terms of their broadband sensitivity, we find that

$$\frac{S}{N} = \left[\frac{S_{gw}^2}{h_{b,bar}^2\, h_{b,int}^2} \frac{T}{\tau_b^3}\right]^{1/4} . \tag{6}$$

Remarkably, *this is independent of the bandwidth B of the bar.* It depends only on the crude "burst" sensitivities of the bar and interferometer. Before we discuss this result, it is instructive to convert this to an energy sensitivity, in terms of the fraction Ω_{gw} of the cosmological closure density per unit logarithmic frequency interval[1]:

$$\begin{aligned} \Omega gw &= 1.5 \times 10^{-5} \left(\frac{f}{1\ \mathrm{kHz}}\right)^2 \left(\frac{h_b,\ int}{10^{-21}}\right) \left(\frac{h_b,\ bar}{10^{-21}}\right) \\ &\quad \times \left(\frac{T}{10^7\ \mathrm{s}}\right)^{-1/2} \left(\frac{H_0}{100\ \mathrm{km\ s^{-1}\ Mpc^{-1}}}\right)^{-2} . \end{aligned} \tag{7}$$

This is an interesting sensitivity. Present observations on millisecond pulsars[14] constrain the gravitational wave background to $\Omega_{gw} \sim 10^{-6}$. But this is at very low frequencies, about 1 cycle per 10 years. It would be interesting to have a constraint at a similar level at kHz frequencies. And if detectors can go below this, then the observations get even more interesting.

Equation 7 leads to several conclusions:

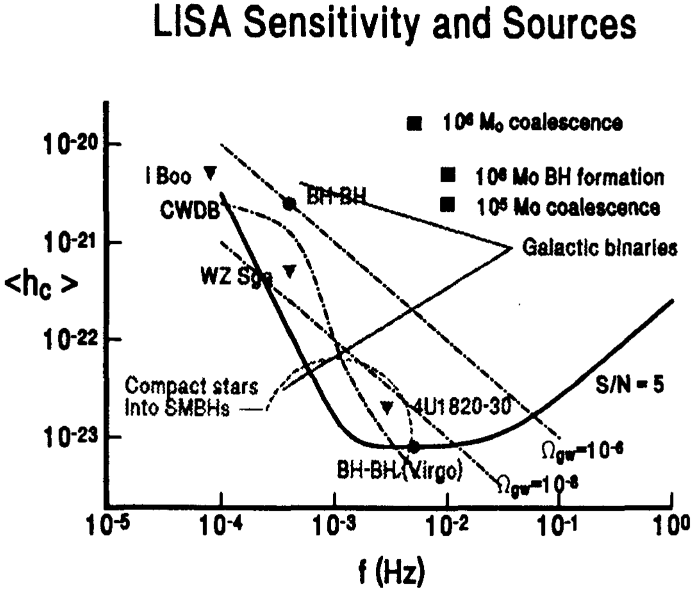

Figure 2: Sensitivity of the proposed LISA space-based detector

1. In a cross-correlation experiment involving one interferometer, the second detector should have the best possible burst sensitivity, regardless of bandwidth. A 10^{-21} bar would be better than a 10^{-20} interferometer, despite its narrower bandwidth.

2. Because interferometers will generally be separated by distances much greater than the reduced wavelength $\bar{\lambda}_{gw} \sim 50(f/1\ \text{kHz})^{-1}$ km. a bar located near an interferometer may well prove more sensitive than two interferometers, even when the bar is less sensitive than the interferometers. On thinks particularly of a future spherical "bar" at LSU and the second LIGO detector, also in Louisiana, and of a ultra-low-temperature bar in Rome and the VIRGO detector in Pisa. This latter combination might not beat the VIRGO-GEO600 combination, which can operate at lower frequency where the de-correlation distance is larger. More detailed calculations are needed in this case.

3. The dependence on frequency in Equation 7 shows that it is easier to constrain Ω_{gw} at low frequencies than at high frequencies. This favours interferometers.
4. The same sort of calculation shows that two bars do even better. If two bars are identical, then for fixed broadband burst sensitivity, their cross-correlation sensitivity varies *inversely* with their bandwidth, by a factor of roughly $(B_b\tau_b)^{1/2}$. For present-day bars, this is a factor of about 30 in Ω_{gw}. This is a significant improvement, and means that identical bars should be considered for such experiments. However, the really have to be identical: if their narrow bandwidths do not overlap, there will be no correlation. I do not know of any pair of bars in existence today that can do this experiment.

6. Sources for LISA

One of the most remarkable developments in the gravitational wave field in the past year has been the positive reception given to the proposal called LISA that the European Space Agency should launch a space-based interferometer to search for low-frequency gravitational waves. Hough will describe this proposal in detail in his talk at this meeting. I will concentrate here on likely sources at the low frequencies accessible to LISA, namely between 0.1 mHz and 1 Hz. This discussion is guided by the theoretical LISA sensitivity curve, shown in Figure 2.

LISA's sources are essentially all long-lived. For most of them, the sensitivity calculation assumes LISA will observe them for one year. Some of the sources, particularly the coalescences of massive black holes, radiate in the LISA band for shorter times, say 4 months. The sensitivity figures for these sources assume only the realistic lifetime.

By observing sources for a year, LISA can provide directional and polarisation information, even though it is a single detector. As it orbits the Sun, the pattern of Doppler shifts in the signal induced by its orbital motion depends on the position of the source in the sky. For weak sources, this provides no resolution below about 1 mHz; and accuracy at the level of a few tens of degrees above this. But for strong sources, such as the massive black-hole coalescences described below, angular accuracy can be as good as 1 arcminute. The orientation of the LISA interferometer plane rotates during the orbit as well, and this produces an amplitude modulation that can be used to determine the polarisation of the signal. This is important information, particularly in binary observations.

I will now describe briefly the kinds of sources LISA should be able to see. This is based largely on the conclusions of the LISA study team. See Hough's article for more detail on the spacecraft and references.

6.1. Binary Star Systems in the Galaxy

Although all 3 known Hulse-Taylor pulsar systems emit gravitational radiation at frequencies somewhat too low for LISA, LISA has much greater range than radio pulsar surveys. The statistical analyses mentioned above[7, 8] suggest that there should be of order 100 neutron-star binaries in the Galaxy within the LISA frequency range, and LISA would be able to see them all. There may be a few black-hole/black-hole systems as well, which will be easily visible. In fact, it is likely that there will be of order one black-hole binary that will be visible from the Virgo cluster.

There are other binaries that ought to be even more plentiful. Some known X-ray binaries and cataclysmic variables are in the range of LISA; in fact, if they were not detected, it would be disastrous for general relativity. There should be a large number of white-dwarf binaries, which are very difficult to detect by other observations. The limits on their population are weak, and it is possible that they will be so plentiful that they will provide a confusion-limited background at low frequencies.

LISA observations of binaries would provide a rich harvest of astrophysical returns. One of the most interesting pieces of information will be the polarisation of the signal. This will tell us the inclination of the orbital plane. For a known binary, whose mass function is known from spectroscopic observations, and whose primary mass is estimated from models, then the inclination will determine the mass of the secondary. Then the intrinsic amplitude of the gravitational waves from the system will determine the distance to the binary. This extra information will be crucial for modelling such systems.

6.2. Stochastic Background of Gravitational Waves

Cosmic strings should produce gravitational waves in this frequency range as well as at higher frequencies. This would appear in LISA observations as a noise. If LISA is launched with the proposed 6 spacecraft, there will be a limited ability to do a cross-correlation between different arms of the detector, which will allow one to distinguish this noise from instrumental noise. Even if there are only 4 LISA spacecraft, we can identify the stochastic noise if we have confidence in our model of the detector noise. This model can be checked in a number of ways once LISA is operating.

LISA should be able to detect a background down to $\Omega_{gw} = 10^{-8}$.

6.3. Solar g-Modes

LISA would respond to any gravitational perturbations, including near-zone, essentially Newtonian perturbations. Therefore, oscillations of the Sun can in principle produce a signal. This would not be noise: the Sun has well-defined discrete frequencies that LISA would easily resolve. Moreover, they could be associated with the Sun instead of an extra-solar-system source because of their lack of a Doppler shift, but presence of an amplitude modulation.

The measured modes of the Sun are all p-modes, whose dominant restoring force is pressure. These are mostly confined to the outer part of the Sun, outside the convection zone, where the mass density is low. For this reason, the gravitational perturbations from these modes seem to be too weak for LISA to detect. However, the Sun has another family of modes, called g-modes or gravity-modes (analogous to the gravity waves of the Earth's atmosphere). These have their largest amplitudes in the centre of the Sun, where the mass density is high. These modes may well be detectable.

The problem is that g-modes have not yet been detected by conventional means. Their lower frequencies and smaller surface amplitudes make them hard to see from Earth. The SOHO mission, to be launched in 1995, will contain an instrument to search for these modes. Interestingly, the sensitivity of the SOHO instrument seems to be about the same as that of LISA: if SOHO sees these modes, so will LISA. (I am grateful to D. Gough for providing me with these calculations.)

6.4. Black Holes in the Centres of Distant Galaxies

The model that active galactic nuclei contain supermassive black holes has gained wide acceptance among astronomers in the last decade. These holes may have masses up to $10^9 M_\odot$. But active nuclei are rare, and most galaxies may have seen only modest amounts of activity in their past. However, there is growing evidence that ordinary galaxies and perhaps even small dwarf galaxies contain more modest black holes in the mass range 10^5–$10^6 M_\odot$. I shall refer to these holes as massive black holes (by contrast with supermassive ones).

The nearest galaxy is of course our own. The evidence for a black hole of this size in our galaxy is still the subject of debate, but what is clear is that there is a concentration of mass of 10^5–$10^6 M_\odot$ in the central core of the Galaxy, which is called Sagittarius A*. What is not so clear is how compact this is: could it be a star cluster, or must it be a black hole? In this connection, the VLBI observation of Sag A* reported in [15] is very suggestive. It shows an elongated region of radio emission that is reminiscent of pictures of jets seen in QSO's. The scale of the emission region is some tens of AU: the emission is coming from a region the size of our Solar System. If this region contains all the mass, then it must be a black hole.

If black holes of modest mass are abundant, then it may be no coincidence that the Jeans mass at the time of recombination is about $10^6 M_\odot$ as well. This is the smallest mass that can collapse under its own gravitation in the normal matter of the Universe, once matter has become transparent to the cosmic background radiation. If black holes form at this epoch, then it is not unreasonable to expect that they would form in groups, just as ordinary stars do. We would then expect the centres of galaxies to have contained binary black holes.

Some fraction of these binaries will be formed close enough to merge together within a Hubble time, due to gravitational radiation reaction on their orbits, just as coalescing neutron-star binaries do. A further proportion will experience tidal friction from stars near the galactic centre, and this will bring them close enough for radiation reaction to bring them together.

A binary merger of two black holes is the strongest source we anticipate for LISA.

A merger at a redshift of 1 would have an amplitude signal-to-noise ratio of 5000 or so. This is so strong that LISA will be able to give a position to within 1 minute of arc, which could allow the galaxy cluster containing the merger to be identified optically, even at $z = 1$. The combination of the redshift and the distance to the source provided by LISA would allow a determination of the Hubble constant, and even more exciting, of the deceleration parameter of the Universe. This is the most exciting prize of the LISA project.

Even without identifications, the strong signal from such a binary, and the fact that the merger is governed entirely by gravitational theory and not by very complicated processed in fluid dynamics and radiative transfer, means that black-hole merger observations can provide a critical test of general relativity. We do not have direct observations of dynamical gravity in strong fields, and this is one place where corrections to gravity from semi-classical fields associated with the graviton in grand-unified field theories might be seen.

Black holes in galactic centres should also occasionally swallow up stars. While main-sequence stars and giants are so large that they will be torn apart by tidal forces before they reach the horizon, neutron stars will remain essentially point particles that follow very complex orbits until they finally fall into the hole. These are not easy to model exactly, but with approximate matched filters that follow portions of the orbit, it should be possible to see these event at redshifts of 1 or so. They would be much more plentiful than black-hole mergers. But they are vulnerable to obscuration by backgrounds: a significant cosmological background or a significant white-dwarf binary background could make these impossible to see.

7. Conclusion and Summary

The field of gravitational wave detection has never looked healthier to me. Although prototype bars and interferometers have not yet made direct detections — we would have been surprised if they had — there are interferometric detectors now being constructed that have every likelihood of doing so. recent designs for spherical solid detectors give narrow-band detectors the opportunity to make real detections as well. And the progress that the LISA project has made in gaining credibility for space-based interferometers is much greater than its proponents expected when they began the proposal.

Against this background, it is heartening that much of the theoretical and astrophysical work on possible gravitational wave sources is heading in the direction of increasing the strength and number of such sources. We have learned nothing in the last few years that reduces the number of sources we expect to see. Instead, we have learned much that increases the chances that supernova explosions are significantly non-spherical, and that neutron-star coalescences may be more frequent than we thought. If we can open up the low-frequency spectrum from space, we may see in the mergers of two black holes some of the most dramatic and informative events that gravitational wave astronomy might ever reveal.

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