Comparison between numerical relativity and a new class of post-Newtonian gravitational-wave phase evolutions: The nonspinning equal-mass case

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We compare the phase evolution of equal-mass nonspinning black-hole binaries from numerical relativity (NR) simulations with post-Newtonian (PN) results obtained from three PN approximants: the TaylorT1 and T4 approximants, for which NR-PN comparisons have already been performed in the literature, and the recently proposed approximant TaylorEt. The accumulated phase disagreement between NR and PN results over the frequency range $M\omega = 0.0455$ to $M\omega = 0.1$ is greater for TaylorEt than either T1 or T4, but has the attractive property of decreasing monotonically as the PN order is increased.

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I. INTRODUCTION

The current interferometric gravitational-wave detectors [1–3] have reached design sensitivity, and have finished taking data in the S5 science run. Gravitational waves from coalescing black-hole binaries will be among the strongest that one hopes to find in the detector data, and data analysts are searching for them by performing matched filtering against template banks of theoretical waveforms. A broad class of theoretical templates can be produced using post-Newtonian (PN) approximation techniques, which are expected to be valid during a binary's slow inspiral.

There exist several prescriptions to compute gravitational-wave (GW) templates at different orders of a PN expansion. Each prescription, termed a PN approximant, provides a slightly different GW phase and hence frequency evolution. For the purpose of GW data analysis, it is important to know which approach, at which PN order, best approximates the true phase evolution of GW signals from astrophysical black-hole binaries. If PN expansions had simple convergence properties and could be taken to arbitrarily high order, one would expect all approaches to converge to the same (presumably correct) result. However, PN calculations do not currently go beyond 3.5PN order, and the convergence properties are far from clear. By 3.5PN order, we mean all the corrections up to the relative order $x^{7/2}$, where $x = (M\omega_b)^{2/3}$, M and ω_b are the binary's total mass and orbital angular frequency. If we consider the number of GW cycles in a given frequency window, for example, the number usually does not change monotonically as the PN order is increased.

One way to check the physical accuracy of the PN results is to compare with fully general-relativistic numerical simulations over the last several orbits of a binary's evolution. Recent breakthroughs in numerical relativity (NR), reported in Refs. [4–6], have made it possible to simulate many orbits before merger [7–9], with the largest number of orbits currently achieved being about 15 [10]. These simulations allow a detailed comparison with various PN approximate GW phase evolutions. Recent studies have shown that standard PN approximants give good phase agreement with NR results up to a few orbits before merger [7,9,10]; for example, for a nonspinning equal-mass binary, the accumulated phase disagreement between NR and PN results that use the TaylorT1 approximant, for the seven orbits before the GW frequency reaches $M\omega = 0.1$ (about two orbits before merger), is less than 1 rad.

A number of PN GW template families have been proposed for use in GW searches (see, for example, [11]), and have been implemented in the LSC Algorithms Library (LAL) [12] for that purpose. We focus here on those based on the Taylor approximants. These approximants model GWs from compact binaries inspiraling due to radiation reaction, and the phase evolution equations are expanded in terms of x. Recently one of us proposed a new class of templates [13] in which the phase evolution $d\phi/dt$ is now given by a PN expansion in terms of the binding energy (see Sec. III for more details). Following the LAL terminology, we refer to this new approximant as TaylorEt. The TaylorEt template has the attractive feature that the number of accumulated GW cycles changes monotonically as we increase the reactive PN order, which is not true for the other Taylor approximants. Further, TaylorEt templates have the same computational cost as, for example, the TaylorT1 and T4 approximants. Additionally, it has been recently demonstrated, while restricting radiation reaction to dominant quadrupole contributions, that the TaylorEt templates are more efficient than TaylorT1 and T4 in capturing GWs from inspiraling compact binaries having orbital eccentricity [14]. Therefore, it is interesting to compare the GW phase evolution under the TaylorEt prescription at various reactive PN orders with the GW phase evolution in numerical simulations. This is what we pursue in this paper.

In this investigation, we compare the GW phase evolution in numerical simulations of equal-mass nonspinning black-hole binaries that last about nine orbits before merger with its counterparts obtainable from TaylorT1, T4 and Et prescriptions at various PN orders. It is important to emphasize that our NR simulations have low initial orbital eccentricity $e \sim 0.0016$ and an accumulated numerical uncertainty of less than 0.25 radians in GW phase evolution. We observe that the accumulated GW phase difference between TaylorT1, T4, and NR oscillates as we increase the reactive PN order from 2PN to 3.5PN in steps of 0.5PN order. However, differences in accumulated GW phases between TaylorEt and NR descriptions decrease monotonically as we increase the reactive phase evolution from 2PN to 3.5PN order in steps of 0.5PN order. This implies that the least GW phase difference associated with TaylorEt occurs at 3.5PN order and it is $\Delta\phi = 1.18$ radians. The observed convergence of the TaylorEt GW phase evolutions towards their NR counterpart and the tolerable accumulated GW phase difference for TaylorEt at 3.5PN order are the two main conclusions of the present paper.

In Sec. II we briefly describe the numerical methods used to produce the NR waveforms. Section III details the construction of different PN prescriptions for the GW phase evolution of the inspiral of an equal-mass binary modeled by nonspinning point-particles. Section IV explains how we make contact between NR and various PN descriptions for GW phase evolutions, and we display our results and provide explanations for our observations. Concluding remarks and future directions are given in Sec. V.

II. NUMERICAL METHODS AND WAVEFORMS

The numerical simulations were performed with the BAM code [15,16], where fourth-order accurate derivative operators were replaced by sixth-order accurate spatial derivative operators in the bulk, as described in [8]. The code started with black-hole binary puncture initial data [17,18] generated using a pseudospectral code [19], and evolved them with the χ-variant of the moving-puncture [20,21] version of the BSSN [22,23] formulation of the $3 + 1$ Einstein evolution equations. The gravitational waves emitted by the binary were calculated from the Newman-Penrose scalar Ψ_4, using the implementation described in [15].

The set of simulations that we discuss here consists of binaries that begin with a coordinate separation of $D = 12M$. The initial momenta are chosen according the prescription given in [24], which lead to inspiral with a minimal eccentricity of $e < 0.0016$. For comparison, a second set of simulations uses initial momenta that lead to a larger eccentricity of $e \approx 0.008$. Simulations were performed at three resolutions. The results were seen to converge consistent with sixth-order accuracy and were subsequently Richardson extrapolated with respect to resolution. Gravitational waves were extracted at five extraction radii, and the GW amplitude was extrapolated with respect to extraction radius to estimate the GW amplitude as measured at infinity. We use the GW phase as measured at the largest extraction radius ($R_{ex} = 90M$). This procedure is described in detail in [9], where the results of these simulations were first reported.

The resulting gravitational waveform has an uncertainty in the amplitude of less than 2%, and an accumulated uncertainty in the phase of less than 0.25 radians.

In the following sections we will focus on comparing the GW phase with that calculated by PN methods. In making comparisons between two waveforms, we make a relative time and phase shift so that the phases and frequencies agree at a given frequency, which we choose to be $M\omega = 0.1$. There will be an uncertainty in determining the matching point in the numerical waveforms due to their nonzero eccentricity. This uncertainty would be large if we matched at low frequencies (i.e., at much earlier times in the numerical waveform), which we did for comparison purposes in [9], but at $M\omega = 0.1$ this is not a dominant source of error. A heuristic calculation was performed in [10], which would suggest that the phase uncertainty due to eccentricity is around 0.1 radians. However, Figure 15 in [9] provides a comparison of the phase from actual simulations with differing eccentricities, and indicates that the phase error due to eccentricity is as much as a factor of 4 lower than that suggested by Eqs. (57) and (60) in [10]. We would therefore place the phase error due to eccentricity conservatively at less than 0.05 radians, and this is absorbed into our overall error estimate of 0.25 radians.

We will now summarize the GW phase as predicted by a selection of PN prescriptions.

III. VARIOUS PRESCRIPTIONS FOR GW PHASE EVOLUTIONS IN PN RELATIVITY

During the inspiral of a compact binary, the associated temporal GW phase evolution can be accurately modeled using the PN approximation to General Relativity. In this approximation, it is customary to consider inspiraling astrophysical compact binaries undergoing adiabatic inspiral along circular orbits due to the emission of gravitational waves. In recent years a number of computational efforts provided four particularly valuable PN expressions that are essential for GW astronomy with inspiraling nonspinning astrophysical compact binaries. These four quantities are the 3PN accurate dynamical (orbital) energy $\mathcal{E}(x)$, expressed as a PN series in terms x, the 3.5PN accurate expression for GW energy luminosity $\mathcal{L}(x)$ and the 3PN amplitude corrected expressions for $h_+(t)$ and $h_\times(t)$, written in terms of the orbital phase ϕ and x [25–28].

GW data analysis groups focusing on inspiraling compact binaries employ these inputs to construct various types of search template. In this paper, as mentioned earlier, we will first consider the TaylorT1 and T4 approximants. These two template families employ the following expression for the restricted PN waveform

$$h(t) \propto x(t) \cos 2\phi(t), \tag{1}$$

where the proportionality constant may be set to unity for nonspinning compact binaries. At a given PN order, the two families provide two slightly different ways to compute $x(t)$ and $\phi(t)$. To obtain the GW phase evolution $\phi(t)$ in the TaylorT1 approximant, one numerically solves the following two differential equations:

$$\frac{d\phi(t)}{dt} \equiv \omega_b(t) = \frac{c^3}{GM} x^{3/2}, \tag{2a}$$

$$\frac{dx(t)}{dt} = -\mathcal{L}(x)\left(\frac{d\mathcal{E}}{dx}\right)^{-1}, \tag{2b}$$

and in this section, we do not employ geometrized units. This implies that to construct TaylorT1 3.5PN search templates, one needs to use 3.5PN accurate $\mathcal{L}(x)$ and 3PN accurate $\mathcal{E}(x)$, respectively. The explicit expressions for these quantities, in the case of equal-mass compact binaries, read

$$\mathcal{L}(x) = \frac{2c^5}{5G} x^5 \left\{ 1 - \frac{373}{84} x + 4\pi x^{3/2} - \frac{59}{567} x^2 - \frac{767}{42} \pi x^{5/2} + \left[\frac{18\,608\,019\,757}{209\,563\,200} + \frac{355}{64}\pi^2 - \frac{1712}{105}\gamma - \frac{1712}{105}\ln(4\sqrt{x}) \right] x^3 + \frac{16\,655}{6048}\pi x^{7/2} \right\}, \tag{3a}$$

$$\mathcal{E}(x) = -\frac{Mc^2}{8} x \left\{ 1 - \frac{37}{48} x - \frac{1069}{384} x^2 + \left[\frac{1\,427\,365}{331\,776} - \frac{205}{384}\pi^2 \right] x^3 \right\}, \tag{3b}$$

where γ is the Euler gamma.

An alternative PN approximant, TaylorT4, has recently been introduced [10]. The TaylorT4 approximant is obtained by Taylor expanding the right-hand side of Eq. (2b) for dx/dt and truncating it at the appropriate reactive PN order. Therefore, to construct TaylorT4 3.5PN search templates, the following set of differential equations are numerically integrated:

$$\frac{d\phi(t)}{dt} \equiv \omega_b(t) = \frac{c^3}{GM} x^{3/2}, \tag{4a}$$

$$\frac{dx(t)}{dt} = \frac{16c^3}{5GM} x^5 \left\{ 1 - \frac{487}{168} x + 4\pi x^{3/2} + \frac{274\,229}{72\,576} x^2 - \frac{254}{21}\pi x^{5/2} + \left[\frac{178\,384\,023\,737}{3\,353\,011\,200} - \frac{1712}{105}\gamma + \frac{1475}{192}\pi^2 - \frac{856}{105}\ln(16x) \right] x^3 + \frac{3310}{189}\pi x^{7/2} \right\}. \tag{4b}$$

The Taylor T1 approximant has been compared with several sets of numerical simulations [9,10] and found to agree with the NR phase to within 1 rad over the GW frequency range $M\omega = 0.05$ and $M\omega = 0.1$, when matched at $M\omega = 0.1$. (Note that the GW frequency is related to the binary's orbital frequency by roughly $\omega = 2\omega_b$.) The Taylor T4 approximant has been found to agree within 0.06 radians when compared in the same way [10].

Let us now describe the TaylorEt approximant. The restricted PN waveform reads

$$h(t) \propto \mathcal{E}(t) \cos 2\phi(t). \tag{5}$$

The time evolution for $\phi(t)$ and $\mathcal{E}(t)$ are obtained by solving the following coupled differential equations.

$$\frac{d\phi(t)}{dt} \equiv \omega_b(t) = \frac{c^3}{Gm} \xi^{3/2} \left\{ 1 + \frac{37}{32}\xi + \frac{12\,659}{2048}\xi^2 + \left[\frac{205}{256}\pi^2 + \frac{3\,016\,715}{196\,608} \right] \xi^3 \right\}, \tag{6a}$$

$$\frac{d\xi(t)}{dt} = \frac{16c^3}{5GM} \xi^5 \left\{ 1 - \frac{197}{336}\xi + 4\pi \xi^{3/2} + \frac{374\,615}{72\,576}\xi^2 + \frac{299}{168}\pi \xi^{5/2} + \left[\frac{3155}{384}\pi^2 - \frac{1712}{105}\ln(4\sqrt{\xi}) + \frac{4\,324\,127\,729}{82\,790\,400} - \frac{1712}{105}\gamma \right] \xi^3 + \frac{4\,155\,131}{96\,768}\pi \xi^{7/2} \right\}. \tag{6b}$$

where $\xi = -2\mathcal{E}/\mu c^2$ and μ being the usual reduced mass. In this paper we keep $d\phi/dt$ to 3PN accuracy (and this is the highest PN order available for Eq. (6a) associated with compact binaries in PN accurate circular orbits). The values of ξ corresponding to any initial and final GW frequencies can be numerically evaluated using the right-hand side of Eq. (6a). This is possible due to the fact that for GWs from compact binaries, having negligible orbital eccentricities, the frequency of the dominant harmonic is $f_{\rm GW} \equiv \omega_b/2$.

At first sight the only difference between TaylorT1/T4 and Et is a different choice of expansion variable. This in itself is interesting to consider: we would like to verify that different (valid) choices of expansion variable do not dramatically change the predictions from PN approximants. In addition, however, the TaylorEt approximant can be easily generalized to eccentric binaries, and indeed the lower order terms in Eq. (6a) are responsible for precession of an eccentric binary.

It should be noted that GW phase and frequency evolutions are prescribed in certain parametric PN accurate ways in the TaylorEt approximant. This is achieved in the TaylorEt approach by prescribing $d\phi/dt$, governing the PN accurate conservative orbital phase evolution, in terms of $\mathcal{E}$ and then numerically imposing secular changes in $\mathcal{E}$, due to the emission of GWs, with the help of far-zone GW energy flux, expressed in terms of $\mathcal{E}$.

In the next section, we compare GW phase evolutions predicted by NR simulations and TaylorT1, T4, and Et approximants at various PN orders in a given GW frequency window.

IV. MAKING CONTACT BETWEEN NR AND PN GW PHASE EVOLUTIONS

In Fig. 1, we plot GW phase differences, with respect to NR simulations in a given GW frequency window, at four post-Newtonian orders, namely, 2PN, 2.5PN, 3PN, and 3.5PN, associated with the three different PN approximants described earlier. The GW frequency window we employ is between $M\omega = 0.0455$ and $M\omega = 0.1$, and we line up the NR and PN GW phases and frequencies at $M\omega = 0.1$. The top left panel refers to TaylorT1 and we observe the usual fluctuating differences in GW phases with respect to the PN order. The top right panel shows a comparison with the TaylorT4 approximant. The good agreement at 3.5PN order that was reported in [10] is clearly visible. Again, we observe that the PN-NR phase disagreement fluctuates with respect to PN order. Further, substantially different GW phase evolutions in comparison with their NR counterpart for TaylorT1 and T4 at 2.5PN order are also observed.

In the bottom panel, we focus our attention on the TaylorEt approximant. When we increase PN accuracy of the reactive dynamics, given by Eq. (6b), from 2PN to 3.5PN in steps of 0.5PN, the GW phase difference, i.e. $\phi_{\rm PN} - \phi_{\rm NR}$, decreases in a monotonic manner. This implies that $\phi_{\rm PN} - \phi_{\rm NR}$ has the lowest value at the 3.5PN order, and is given by $\Delta\phi = -1.18$ radians. This GW phase difference at 3.5PN order is definitely more than its counterparts arising from TaylorT1 and T4 ($\Delta\phi = 0.6$ and 0.05, respectively). However, it is intriguing that the TaylorEt approximant seems to display monotonic convergence towards the NR phase. We have checked that monotonic convergence to NR is independent of the choice of our matching frequency.

It is possible to explain, using PN based arguments, the comparatively large gap one finds between 2.5 and 3PN orders in the bottom panel of Fig. 1. Recall that the conservative phase evolution in the TaylorEt approximant is 3PN accurate [in other words, we always use 3PN accurate expression for $d\phi/dt$, given by Eq. (6a)] and this requires a complete knowledge about the 3PN accurate conservative orbital dynamics. Further, it should be clear that the reactive GW phase evolution in TaylorEt is governed by Eq. (6b) that prescribes $\xi(t)$. It is not difficult to realize that to compute $d\xi/dt$ at 2PN and 2.5PN orders, one only needs to know conservative orbital dynamics to 2PN order. However, computations that lead to $d\xi/dt$ at 3PN and 3.5PN orders require conservative orbital dynamics to 3PN order. We expect that the change in the conservative dynamics from 2PN to 3PN order while computing $d\xi/dt$ is the dominant reason for the observed gap in $\phi_{\rm PN} - \phi_{\rm NR}$ between 2.5PN and 3PN orders in the bottom panel of Fig. 1.

We provide Fig. 2 to argue that monotonic convergence to the "exact" GW phase, exhibited by the TaylorEt templates is rather insensitive to small numerical eccentricities present in NR evolutions. We obtained Fig. 2 by employing NR puncture evolutions having an initial eccentricity $e \approx 0.008$. The oscillatory nature of $\phi_{\rm PN} - \phi_{\rm NR}$ clearly demonstrates that we are indeed dealing with NR simulations that possess some tiny orbital eccentricity. It is also evident from Fig. 2 that $\phi_{\rm PN} - \phi_{\rm NR}$ at any given PN order is not that substantially different from what is given in the bottom panel of Fig. 2; the accumulated phase difference was $\Delta\phi = -1.18$ radians for the data with negligible eccentricity, and is $\Delta\phi = -1.16$ for the more eccentric data.

Let us explore another unique feature of TaylorEt approximant relevant for making contact with NR-based

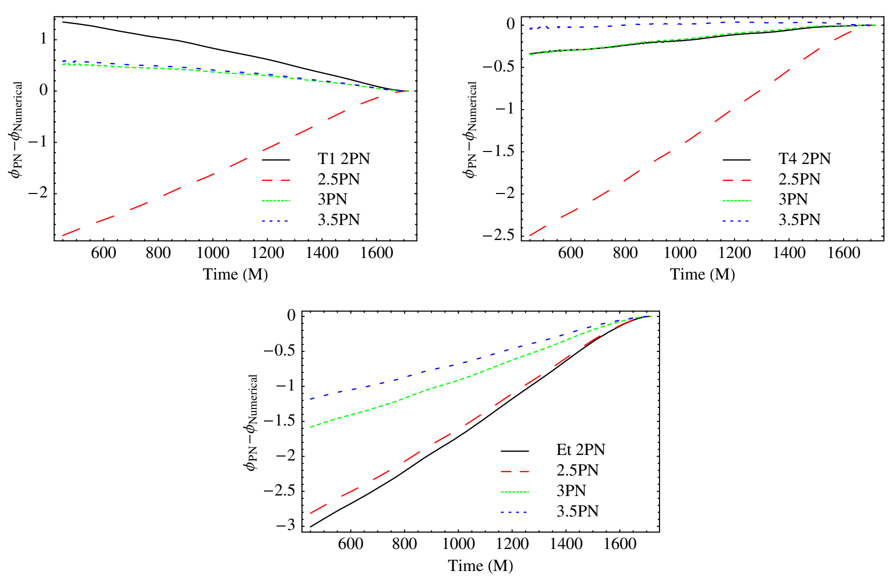

FIG. 1 (color online). Accumulated phase disagreement between NR and PN evolutions for TaylorT1, T4, and Et approximants at 2PN, 2.5PN, 3PN, and 3.5PN reactive orders.

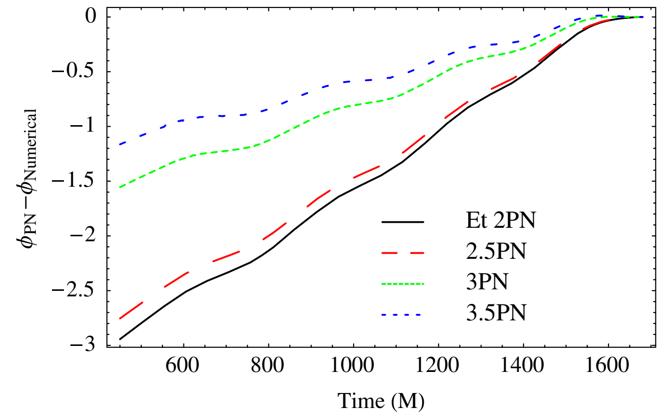

FIG. 2 (color online). Accumulated phase disagreement between NR results and the PN approximant TaylorEt, as in the lower panel of Fig. 1, with the difference that we now use numerical data from a simulation with slightly larger eccentricity, $e \approx 0.008$.

black-hole binary evolutions. A close inspection of Eqs. (6) defining TaylorEt GW phase evolution reveals that if one can provide bounding values for ξ, it is possible to obtain the associated $\phi(t)$ at various PN orders.

One can estimate the binding energy on one time slice of a numerical black-hole spacetime by calculating the difference between the total energy of the spacetime, the Arnowitt-Deser-Misner (ADM) mass, and the total mass of the two black holes. This estimate of the binding energy was introduced in [29], and is usually performed on the initial time slice, and as such includes the energy in the "junk" radiation associated with the standard initial-data choices. However, the junk radiation quickly radiates away, and then we may make a potentially more reliable estimate of the binding energy by calculating

$$E_b = E_{\rm ADM} - M_1 - M_2 - E_{\rm rad}, \tag{7}$$

assuming that one has accurately measured the energy lost in gravitational-radiation emission, $E_{\rm rad}$. We find that we can estimate the radiated energy with sufficient accuracy to place an uncertainty on the binding energy of less than 2%.

When we perform a matching with respect to binding energy, such that the binding energy and phase are equal at some time (in Fig. 3, $E_b = -0.013\,83M$ for both NR and PN waveforms at the matching time), the GW frequencies of the NR and PN waveforms are *not* the same at that time. Therefore the time derivative of the phase disagreement is nonzero, and the phase disagreement immediately grows linearly as we move backwards in time.

The curves in Fig. 3 possess a turning point that makes an estimate of the accumulated phase error ambiguous. However, we should point out that this ambiguity exists in all NR-PN phase comparisons, and this is clear in the figures shown in Refs. [9,10]. It is always possible to

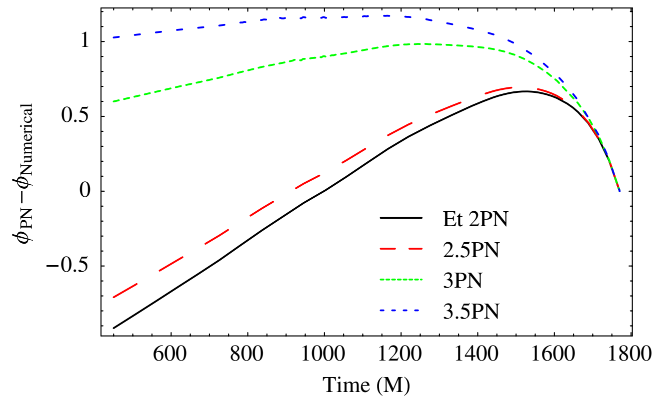

FIG. 3 (color online). The same comparison as in Fig. 2, but we now match the phase and *binding energy* such that $E_b = -0.013\,83M$ at the matching time.

perform a matching at a different time (and therefore a different frequency or, in this case, binding energy), and to find a different accumulated phase disagreement, depending on the new location of the turning point. The comparison between the NR and PN phases is cleanest when a matching point is chosen such that there is no turning point, as in the analysis presented earlier and demonstrated in Fig. 1. Despite these ambiguities, we once again see only monotonic changes in the accumulated phase error when comparing NR results with the TaylorEt approximant.

V. DISCUSSION

In this paper, we have compared GW phase evolutions associated with nonspinning equal-mass NR-based black-hole binary inspiral lasting nine orbits with three different prescriptions based on the TaylorT1, T4, and Et approximants. We verified that GW phase evolution prescribed by TaylorT4 at 3.5PN order agrees very well with its NR counterpart. Further, $\phi_{\rm PN} - \phi_{\rm NR}$ associated with TaylorT1 and T4 approximants fluctuates as we increase PN order from 2PN to 3.5PN in steps of 0.5PN. However, the recently introduced TaylorEt approximant displays an intriguing feature, namely, $\phi_{\rm PN} - \phi_{\rm NR}$ decreases in a monotonic fashion as we increase the PN order responsible for reactive GW phase evolution. We also infer that $\phi_{\rm PN} - \phi_{\rm NR}$ associated with the TaylorEt approximant at 3.5PN order should be tolerable for the purpose of GW detection. Figure 4 summarizes our NR versus PN comparisons.

The monotonic convergence to the fully general-relativistic GW phase evolution, exhibited by the TaylorEt templates, along with its other attributes listed earlier, makes it an interesting candidate for GW templates that model inspiral, or for producing hybrid PN-NR waveforms as in [30–32].

The observation that TaylorEt templates should be more efficient in capturing GWs from compact binaries having small orbital eccentricities in comparison with TaylorT1 and T4, demonstrated in Ref. [14] while restricting the

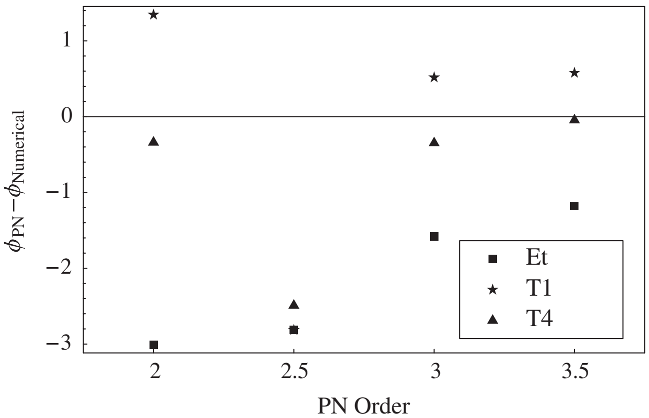

FIG. 4. Accumulated phase disagreement between NR and PN results, for each of the three approximants, TaylorT1, TaylorT4, and TaylorEt at 2PN, 2.5PN, 3PN, 3.5PN orders. At 2.5PN the TaylorT1 and TaylorEt points are on top of each other. Note that the disagreement between NR and TaylorEt decreases monotonically as the reactive PN order is increased.

radiation reaction to dominant contributions, makes it a promising candidate to search for GWs using data from GEO-LIGO and VIRGO. Data analysis implications of GW polarizations $h_{\times,+}(t)$, evolving under the TaylorEt prescription, relevant for ground and space-based GW interferometers are currently under investigation.

In future work we plan to make similar comparisons involving unequal-mass and spinning binaries.

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