

Spinning strings at one-loop in $\text{AdS}_4 \times \mathbb{P}^3$

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ABSTRACT: We analyze the folded spinning string in $\text{AdS}_4 \times \mathbb{P}^3$ with spin S in AdS_4 and angular momentum J in \mathbb{P}^3 . We calculate the one-loop correction to its energy in the scaling limit of both $\ln S$ and J large with their ratio kept fixed. This result should correspond to the first subleading strong coupling correction to the anomalous dimension of operators of the type $\text{Tr}(D^S(Y^\dagger Y)^J)$ in the dual $\mathcal{N} = 6$ Chern-Simons-matter theory. Our result appears to depart from the predictions for the generalized scaling function found from the all-loop Bethe equations conjectured for this $\text{AdS}_4/\text{CFT}_3$ duality. We comment on the possible origin of this difference.

KEYWORDS: AdS-CFT Correspondence, Integrable Field Theories.

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1. Introduction

The spinning folded string in AdS_5 has played an important role in our quantitative understanding of the AdS/CFT duality. In the large spin limit, the difference between its energy E and spin S scales like $\ln S$ [1]; the proportionality coefficient is the universal scaling function $f(\lambda)$ which provided the first controlled example of an interpolating function between weak and strong coupling. These spin S states are thought to be dual to the operator $\text{tr}(ZD^S Z)$ where D is the light-cone covariant derivative and Z is one of the complex scalar fields of the theory; for such operators the logarithmic scaling has long been known [2–4].

A spinning folded string also exists in sigma models on lower-dimensional AdS spaces, such as $\text{AdS}_4 \times \mathbb{P}^3$; it was pointed out in [5] that in the large spin limit they have similar properties as the AdS_5 state, that is

$$E - S \propto \ln S + \mathcal{O}(S^0) \quad . \tag{1.1}$$

The gauge theory dual to closed string theory on $\text{AdS}_4 \times \mathbb{P}^3$ was recently conjectured to be a certain $\mathcal{N} = 6$ superconformal three-dimensional Chern-Simons theory [5] (see also [6]).

At finite N and k , this $U(N) \times U(N)$ gauge theory is in fact thought to describe the low-energy physics of N M2-branes on $\mathbb{R}^{1,2} \times \mathbb{C}^4/\mathbb{Z}_k$, where k is interpreted as the level of the Chern-Simons theory (for recent discussions on the M2-brane worldvolume theory see e.g. [7–14]); in the large N limit the gravity dual becomes M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$ where the orbifold group lies inside a $U(1)$ subgroup of the $SO(8)$ isometry group of S^7 . This theory also has an 't Hooft limit where both k and N are taken to be large with $\lambda = N/k$ kept fixed. In this limit the size of the circle fiber acted upon by the \mathbb{Z}_k orbifold becomes very small and thus the appropriate description is as type IIA theory on $AdS_4 \times \mathbb{P}^3$. The $\mathcal{N} = 6$ Chern-Simons theory [5] exhibits an $SU(4) \times U(1)$ global symmetry group, the first factor of which is the R-symmetry. In addition to the gauge-fields, it also contains eight bi-fundamental scalar fields Y^I and Y_I^\dagger which transform as $\mathbf{4}_{+1}$ and $\bar{\mathbf{4}}_{-1}$ of $SU(4) \times U(1)$. The representations of the eight fermionic bi-fundamental superpartners follow from the representation of the supercharges; for the M2-brane theory the supercharges transformed as the $\mathbf{8}_c$ representation of the $SO(8)$ R-symmetry and decompose under the commutant of the orbifold action as $\mathbf{6}_0 \oplus \mathbf{1}_2 \oplus \mathbf{1}_{-2}$. It is natural to expect that the spinning folded strings should be dual to single trace gauge invariant operators made of a large number of covariant derivatives and some finite number of other fields.

The twist-two operators $\text{tr}(ZD^S Z)$ of $\mathcal{N} = 4$ SYM theory are not captured by the asymptotic Bethe ansatz. To bypass this problem and, at the same time, to make a cleaner identification between the gauge theory operators and string solutions it is useful to generalize the rotating folded string by adding a further angular momentum J in the compact space. The dual operators $\text{tr}(D^S Z^J)$ belong to the $sl(2)$ sector of the theory. For strings in $AdS_5 \times S^5$ this has been done in [15]. The resulting target space energy, $E(\sqrt{\lambda}, S, J)$, is a nontrivial function of its arguments and may be expanded in different regimes, uncovering and testing various aspects of the gauge and string Bethe ansätze. One can straightforwardly find similar strings moving along an $S^1 \subset \mathbb{P}^3$ with angular momentum J . Invariance under $U(N) \times U(N)$ gauge transformations, and the requirement that the operator be charged only under one Cartan generator of the R-symmetry group suggests that the relevant operators are $\text{tr}(D^S (Y^1 Y_4^\dagger)^J)$.^{1,2}

As for the $\mathcal{N} = 4$ theory, the dilatation operator of the Chern-Simons theory appears to be described by an integrable spin chain at weak coupling [16] (see also [17, 18]). Unlike that of the $\mathcal{N} = 4$ theory this spin chain is alternating due to the presence of fields in the bifundamental representation. Given as J roughly corresponds to the spin-chain length, it is necessary to take it to be large in order to expect an exact Bethe ansatz, which would therefore be asymptotic. The choice of vacuum for the spin chain leaves unbroken a symmetry group similar to that of the spin chain of $\mathcal{N} = 4$ SYM theory. Together with information [19] extracted from a conjectured worldsheet action for strings in $AdS_4 \times \mathbb{P}^3$

¹Here we assigned charges to the fields in the $\mathbf{4}$ of $SU(4)$ such that Y^4 has equal charges under all three Cartan generators while Y^i with $i = 1, 2, 3$ has the same charge as $(Y_4)^\dagger$ under the i -th generator and the charge as Y^4 under the other two generators.

²It is worth noting that two scalar fields together with a covariant derivative can carry the same quantum numbers as a fermion bilinear so that generically such states will mix; with a some care however, it is still possible to identify a closed $sl(2)$ sector.

[20–22], asymptotic Bethe equations have been conjectured in [23] (see also [24]). To leading order in the weak coupling expansion these equations reproduce the results of direct anomalous dimension calculations [16]. Similarly to $\text{AdS}_5 \times \text{S}^5$, the study of the properties of classical string solutions, such as the finite size corrections to their energy, (see [25–30]) may be used to carry out further tests of the Bethe equations.

In this work we will consider the one-loop string corrections to the energy of the spinning folded string in $\text{AdS}_4 \times \mathbb{P}^3$. While the full superstring action on this space is not known, sigma models based on the coset $\text{Osp}(6|4)/\text{SU}(3) \times \text{U}(1) \times \text{SO}(3,1)$ and supplied with an appropriate Wess-Zumino like term [20–22] have been suggested to represent partially κ -gauge fixed Green-Schwarz string actions. Furthermore, it has been shown that these actions are classically integrable suggesting that it may be possible to study this theory using similar methods to the $\text{AdS}_5 \times \text{S}^5$ case. We will however not use these actions. To one-loop order only the quadratic part of the fermion action is necessary and its structure is well-known in terms of the supersymmetric covariant derivative.

After recalling the supergravity background [5] in §2 we proceed in §3 to discuss the spinning string solutions in $\text{AdS}_4 \times \mathbb{P}^3$, some of their scaling limits as well as the expectations for the semiclassical expansions of their energy, all of which are quite analogous to those of spinning strings in $\text{AdS}_5 \times \text{S}^5$. In §4 we find the spectrum of bosonic and fermionic fluctuations around the spinning folded string solution in the scaling limit. In §5 we evaluate the one-loop correction to the target space energy both for strings with $J = 0$ and $J \neq 0$ in the semi-classical scaling limit. We show that the quadratic and logarithmic divergences cancel and extract the one-loop correction to the generalized scaling function. In §6 we discuss the comparison with the Bethe ansatz predictions and discuss some possible future directions.

2. $\text{AdS}_4 \times \mathbb{P}^3$ background

Recently, [5], it was pointed out that the near horizon geometry of M2-branes on a special \mathbb{Z}_k quotient of flat space is, for large values of k , $\text{AdS}_4 \times \mathbb{P}^3$. Taking the standard M2-brane near horizon geometry of $\text{AdS}_4 \times \text{S}^7$ and writing the S^7 as a S^1 fibration over \mathbb{P}^3 the effect of the \mathbb{Z}_k quotient is simply to make the radius of the S^1 smaller by a factor of k . The compactification from eleven to ten dimensions gives rise to a two form flux which is proportional to the Kähler form on the \mathbb{P}^3 and the four form flux is unaffected except that the number of units of flux is reduced by a factor of k . To be more explicit the background fields after the quotient are

$$\begin{aligned}
 ds^2 &= \frac{R^3}{4k} (ds_{\text{AdS}_4}^2 + 4ds_{\mathbb{P}^3}^2) & e^{2\phi} &= \frac{R^3}{k^3} \\
 F_2 &= k J_{\mathbb{P}^3} & F_4 &= \frac{3}{8} R^3 \text{Vol}_{\text{AdS}_4}
 \end{aligned}
 \tag{2.1}$$

Above, the metric and the forms are written in terms of those of spaces of unit radius. For AdS_4 we use global coordinates, (t, ρ, θ, ϕ) and the resulting metric is the standard

$$ds_{\text{AdS}_4}^2 = -\cosh^2 \rho dt^2 + d\rho^2 + \sinh^2 \rho (d\theta^2 + \sin^2 \theta d\phi^2)
 \tag{2.2}$$

and we make use of the parameterization, [31], of the \mathbb{P}^3 geometry in terms of the coordinates $(\zeta_1, \zeta_2, \zeta_3, \tau_1, \tau_2, \tau_3)$,

$$ds_{\mathbb{P}^3}^2 = d\zeta_1^2 + \sin^2 \zeta_1 \left[d\zeta_2^2 + \cos^2 \zeta_1 (d\tau_1 + \sin^2 \zeta_2 (d\tau_2 + \sin^2 \zeta_3 d\tau_3))^2 + \sin^2 \zeta_2 \left(d\zeta_3^2 + \cos^2 \zeta_2 (d\tau_2 + \sin^2 \zeta_3 d\tau_3)^2 + \sin^2 \zeta_3 \cos^2 \zeta_3 d\tau_3^2 \right) \right] \quad (2.3)$$

where we have pulled out an overall factor of $R_{\text{AdS}}^2 = R^3/4k$ with R being the radius of the original $\text{AdS}_4 \times \text{S}^7$ geometry. This expression for the \mathbb{P}^3 metric can be found by iteratively embedding \mathbb{P}^{n-1} in \mathbb{P}^n . The two-form can be written as the exterior derivative, $F_2 = kd\omega$, of a one-form defined locally by

$$\omega = \sin^2 \zeta_1 (d\tau_1 + \sin^2 \zeta_2 (d\tau_2 + \sin^2 \zeta_3 d\tau_3)) . \quad (2.4)$$

In physical coordinates one has:

$$(F_2)_{\mu\nu} = 2 \frac{k^2}{R^3} J_{\mu\nu} \quad (F_4)_{abcd} = 6 \frac{k^2}{R^3} \epsilon_{abcd} \quad (2.5)$$

or

$$e^\phi (F_2)_{\mu\nu} = \frac{1}{R_{\text{AdS}}} J_{\mu\nu} \quad e^\phi (F_4)_{abcd} = \frac{3}{R_{\text{AdS}}} \epsilon_{abcd} \quad (2.6)$$

where J and ϵ are numerical tensors with entries ± 1 and 0 . They are, respectively, the entries of the Kähler form and of the volume form on unit \mathbb{P}^3 and AdS_4 . Finally the ten-dimensional radius of curvature will be related to the 't Hooft coupling by

$$R_{\text{string}}^2 = \frac{R^3}{k} = 2^{5/2} \pi \sqrt{\lambda} . \quad (2.7)$$

We now turn to the study of a particular class of spinning strings in this background.

3. Spinning string solution and scaling limits

Many of the spinning string solutions of Frolov and Tseytlin [15, 32, 33] are again solutions of strings on $\text{AdS}_4 \times \mathbb{P}^3$ and indeed many of their calculations, including that of the quantum correction to the long spinning string, are modified only very slightly. Let us briefly summarize some of the relevant details about spinning strings. We wish to consider folded closed strings that have two non-vanishing charges: one spin, S , in the AdS_4 space and one angular momentum, J , in the compact \mathbb{P}^3 and that are solutions of the equations of motion of the action

$$I = I_{\text{AdS}_4} + I_{\mathbb{P}^3} = \frac{R_{\text{AdS}}^2}{4\pi} \int d\tau d\sigma \sqrt{-h} h^{ab} \left(G_{\mu\nu}^{\text{AdS}} \partial_a X^\mu \partial_b X^\nu + 4G_{\mu\nu}^{\mathbb{P}^3} \partial_a X^\mu \partial_b X^\nu \right) . \quad (3.1)$$

Due to the choice of spins, the solution fits inside an $\text{AdS}_3 \times \text{S}^1$ subspace and it is in fact identical to that of GKP, [1] and further studied in [15], except for a multiplication of the

S^1 angular momentum parameter by $\frac{1}{2}$. This is a consequence of the numerical factor in the second term in the action (3.1).

As in $\text{AdS}_5 \times S^5$, the worldsheet semiclassical expansion about these spinning string solutions is naturally organised as an expansion in $\frac{1}{\sqrt{2\lambda}}$ (which is proportional to the inverse string tension) which keeps fixed the charge densities $\mathcal{S} = \frac{S}{\sqrt{2\lambda}}$ and $\mathcal{J} = \frac{J}{\sqrt{2\lambda}}$. The target space energy of the string is given by

$$E = \sqrt{2\lambda} \mathcal{E} \left(\mathcal{S}, \mathcal{J}, \frac{1}{\sqrt{2\lambda}} \right) = \sqrt{2\lambda} \left[\mathcal{E}_0(\mathcal{S}, \mathcal{J}) + \frac{1}{\sqrt{2\lambda}} \mathcal{E}_1(\mathcal{S}, \mathcal{J}) + \dots \right]. \quad (3.2)$$

Given the complexity of the solution [15] additional limits are useful. We will consider the so-called ‘‘semi-classical scaling’’ or long-string limit of the spinning string solutions, see [15, 34] and also [35],

$$\mathcal{S} \gg \mathcal{J} \gg 1, \quad \text{with } \ell \equiv \frac{\mathcal{J}}{2 \ln \mathcal{S}} \text{ fixed.} \quad (3.3)$$

Since we are interested in the limits $\ln \mathcal{S} \gg \ln \mathcal{J}$ and $\mathcal{S} \gg \sqrt{2\lambda}$ this equivalent to

$$S \gg J \gg 1, \quad \text{with } \ell \approx \frac{J}{2\sqrt{2\lambda} \ln S} \text{ fixed.} \quad (3.4)$$

As discussed at length in [34, 35], in this limit the solution simplifies dramatically becoming homogeneous. Choosing³ the angle φ_3 parametrizing the circle $S^1 \subset \mathbb{P}^3$ as $\varphi_3 = \frac{1}{2}(\tau_1 + \tau_2 + \tau_3)$, the relevant part of the action is given by the metric

$$ds^2 = R_{\text{AdS}}^2 (d\rho^2 - \cosh^2 \rho dt^2 + \sinh^2 \rho d\phi^2 + 4d\varphi_3^2) . \quad (3.6)$$

Then, the solution is just

$$\bar{t} = \kappa\tau \quad \bar{\phi} = \kappa\tau \quad \bar{\rho} = \mu\sigma \quad \bar{\varphi}_3 = \frac{1}{2}\nu\tau \quad \mu = \sqrt{\kappa^2 - \nu^2} ; \quad (3.7)$$

the other $\text{AdS}_4 \times \mathbb{P}^3$ coordinates take constant values, the nonvanishing ones being

$$\bar{\theta} = \frac{\pi}{2}, \quad \bar{\zeta}_1 = \frac{\pi}{4}, \quad \bar{\zeta}_2 = \frac{\pi}{2}, \quad \bar{\zeta}_3 = \frac{\pi}{2} . \quad (3.8)$$

As with all classical solutions, two-dimensional Lorentz invariance is spontaneously broken. As we shall see it turns out to be convenient to express the solution in terms of constant vectors. In this way, Lorentz invariance is apparently preserved (and it would be if one allowed these constant vectors to transform as implied by the indices they carry). In analogy with the spinning string solution in $\text{AdS}_5 \times S^5$, we define the vectors \hat{n} , \tilde{n} and \hat{m}

$$d\bar{t} = \hat{n} \cdot d\boldsymbol{\sigma} \quad d\bar{\phi} = \hat{n} \cdot d\boldsymbol{\sigma} \quad d\bar{\rho} = \tilde{n} \cdot d\boldsymbol{\sigma} \quad d\bar{\varphi}_3 = \frac{1}{2}\hat{m} \cdot d\boldsymbol{\sigma} \quad \boldsymbol{\sigma} = (\sigma^0, \sigma^1) \equiv (\tau, \sigma). \quad (3.9)$$

³There are many different S^1 factors that one may pick inside \mathbb{P}^3 . A particularly useful choice, which leads to the vanishing of some components of the spin connection, may be identified by introducing new coordinates

$$\tau_1 = \varphi_3 - \beta, \quad \tau_2 = \beta - \gamma, \quad \tau_3 = \varphi_3 + \gamma . \quad (3.5)$$

with all the other coordinates set to zero.

The Virasoro constraint relates these vectors as follows:

$$\eta^{ab} \hat{n}_a \hat{n}_b + \eta^{ab} \tilde{n}_a \tilde{n}_b = \eta^{ab} \hat{m}_a \hat{m}_b = -\nu^2 . \quad (3.10)$$

We must also impose periodicity in the σ direction, $\bar{\rho}(\sigma + 2\pi) = \bar{\rho}(\sigma)$, which is satisfied by interpreting the solution (3.7) as a string folded onto itself. The string is thus made of four segments: for, $0 \leq \sigma \leq \frac{\pi}{2}$, $\bar{\rho}$ increases from 0 to its maximum ρ_0 , while for $\frac{\pi}{2} \leq \sigma \leq \pi$ it decreases from its maximum value back to zero and then repeats. The relation between the parameters of the solution, κ , μ and ν , is a consequence of the Virasoro constraint. We note that for the above solution, being in the scaling limit (3.3), κ and μ are both large while $\ell = \frac{\nu}{\mu}$ is kept fixed. This can be seen clearly by relating the parameters of the solution to the global charges of the string which are given by, $E = \sqrt{2\lambda} \mathcal{E}$ etc, with

$$\begin{aligned} \mathcal{E} &= \int_0^{2\pi} d\sigma \frac{\kappa}{2} \cosh^2 \bar{\rho}, & \mathcal{S} &= \int_0^{2\pi} d\sigma \frac{\kappa}{2} \sinh^2 \bar{\rho}, \\ \mathcal{J} &= \int_0^{2\pi} d\sigma \nu. \end{aligned} \quad (3.11)$$

We thus have

$$\mu = \frac{1}{\pi} \ln \mathcal{S}, \quad \mu \gg 1, \quad \ell = \frac{\nu}{\mu} = \text{fixed}. \quad (3.12)$$

If we rescale σ by μ we get $\rho = \sigma$ but now the worldsheet has length $L = 2\pi\mu \sim \ln S \gg 1$ and in the strict $L \rightarrow \infty$ limit the closed string can be thought of as two infinite overlapping open strings. In this limit we can neglect all effects of the string end points where from the closed string point of view the worldsheet curvature becomes infinite. In the scaling limit (3.3) we have $\mathcal{E} = \mathcal{S} + \kappa \pi$ and thus to leading order

$$\mathcal{E}_0 - \mathcal{S} = \mu\pi \sqrt{1 + \ell^2} = \ln S \sqrt{1 + \frac{\mathcal{J}^2}{4 \ln^2 S}} \quad (3.13)$$

or using the fact that $\mathcal{S} \gg \mathcal{J}$ and $\frac{\mathcal{S}}{\sqrt{2\lambda}} \gg 1$

$$\begin{aligned} E_0 - S &= \sqrt{2\lambda} \ln S \sqrt{1 + \frac{\mathcal{J}^2}{8\lambda \ln^2 S}} \\ &= \sqrt{2\lambda} f_0(\ell) \ln S . \end{aligned} \quad (3.14)$$

We can of course consider the limit in which the angular momentum in the compact space is vanishing, or more precisely the limit $\frac{\mathcal{J}}{\ln S} \ll 1$, the ‘‘semi-classical scaling small’’ limit. In this limit at leading order $E_0 - S = \sqrt{2\lambda} \ln S$ which is the result from [5].

Our aim here is to extend this result to include the next-to-leading order correction to the spinning string energy which, as we shall explicitly see, takes the form

$$E_1 = f_1(\ell) \ln S + \dots . \quad (3.15)$$

Thus, just as for the $\text{AdS}_5 \times \text{S}^5$ string, it appears that the strong coupling expansion in the scaling limit can be organised as

$$E - S = \sqrt{2\lambda} f(\ell, \lambda) \ln S + \dots \quad (3.16)$$

and the function, $f(\ell, \lambda)$ can be expanded in inverse powers of $\sqrt{2\lambda}$ to give the coefficients $f_0(\ell)$, $f_1(\ell)$, etc or alternatively one can first expand in powers of ℓ

$$f(\ell, \lambda) = f(\lambda) + \ell^2 q(\ell, \lambda) + \ell^4 p(\ell, \lambda) + \dots \tag{3.17}$$

The function $f(\lambda)$ is the three-dimensional analogue of the universal scaling function $f(\lambda)$ of $\mathcal{N} = 4$ super-Yang-Mills in four dimensions. Similarly to that case, we expect that the functions $q(\ell, \lambda)$ and $p(\ell, \lambda)$ exhibit logarithmic dependence on ℓ in the string coupling expansion.

It is perhaps worth mentioning that the relationship between GKP spinning strings [1] and the open strings dual to light-like Wilson loops with a cusp, [36], that is known to exist in AdS_5 , persists in this context at least at the level of the classical worldsheet. The argument, [37], that in the scaling limit, after an analytic continuation combined with the use of the AdS isometries, these two string solutions correspond to the same minimal surface is essentially unchanged. Thus we expect the anomalous dimension of twist-two operators and the cusp anomaly to be equal also in the dual three-dimensional Chern-Simons theory. Their common value should define the scaling function $f_{\text{CS}}(\lambda)$.⁴ This equivalence for the $\mathcal{N} = 4$ theory was proven in weak coupling perturbation theory [2–4] and in addition to the arguments cited above has been partially confirmed by direct calculation [36, 15, 37, 38]. It is also worthwhile mentioning that the same scaling function $f(\lambda)$ governs the IR asymptotics of the gluon amplitude in the $\mathcal{N} = 4$ theory [39–45]. Furthermore for the four-point gluon amplitude it determines the finite part of the exponentiated all-loop expression found in [45, 46]. In the context of the AdS/CFT correspondence the same functional dependence for the scattering amplitude was found at strong coupling by [47]. In large part this is entirely determined by the symmetries of the problem [48, 49]. For AdS_4 we can, at least at strong coupling, formally find a similar relation though the interpretation, which makes use of several T-duality like transformations, is perhaps less clear.

4. Fluctuation spectrum

4.1 Bosonic action to quadratic order

In this section we calculate the spectrum of bosonic quantum fluctuations about the spinning string solutions, at least in the homogeneous scaling limit. In this we will again follow very closely [15, 34] and so we will not belabor the details - the calculations are essentially identical though with one less transverse degree of freedom in the AdS space and one more in the slightly more complicated \mathbb{P}^3 space. The fluctuations about the classical spinning string solution in the AdS_4 space are

$$t = \hat{n} \cdot \boldsymbol{\sigma} + \frac{\tilde{t}}{\sqrt{\lambda^{\frac{1}{4}}}}, \quad \rho = \tilde{n} \cdot \boldsymbol{\sigma} + \frac{\tilde{\rho}}{\sqrt{\lambda^{\frac{1}{4}}}}, \quad \theta = \frac{\pi}{2} + \frac{\tilde{\theta}}{\sqrt{\lambda^{\frac{1}{4}}}}, \quad \phi = \hat{n} \cdot \boldsymbol{\sigma} + \frac{\tilde{\phi}}{\sqrt{\lambda^{\frac{1}{4}}}} \tag{4.1}$$

⁴The coordinate transformations relating the spinning folded string to the Wilson line with a cusp can also be carried out in the presence of nonvanishing angular momentum on S^5 .

In the above we have used as our expansion parameter⁵ $\tilde{\lambda} = 2\pi^2\lambda$. The bosonic action quadratic in fluctuations in the AdS₄ space becomes

$$I_{\text{AdS}_4} = -\frac{1}{4\pi} \int d^2\sigma \left[(\partial\rho)^2 - \cosh^2 \bar{\rho} (\partial t)^2 + [(\partial\phi)^2 + (\partial\theta)^2 - \hat{n} \cdot \hat{n} \theta^2] \sinh^2 \bar{\rho} \right. \\ \left. - 2(\hat{n} \cdot \partial t - \hat{n} \cdot \partial\theta) \kappa \rho \sinh(2\bar{\rho}) \right] \quad (4.2)$$

where we have dropped the tildes. To eliminate the explicit dependence on $\bar{\rho}$ it is useful to redefine the fields as

$$\hat{t} = \cosh \bar{\rho} t, \quad \hat{\theta} = \sinh \bar{\rho} \theta, \quad \hat{\phi} = \sinh \bar{\rho} \phi, \quad \hat{\rho} = \rho \quad (4.3)$$

and do a further rotation in the $(\hat{t}, \hat{\phi})$ plane

$$\chi = \hat{\phi} \cosh \bar{\rho} - \hat{t} \sinh \bar{\rho}, \quad \zeta = -\hat{\phi} \sinh \bar{\rho} + \hat{t} \cosh \bar{\rho} \quad (4.4)$$

after which the action becomes

$$I_{\text{AdS}_4} = -\frac{1}{4\pi} \int d^2\sigma \left[-(\partial\zeta)^2 + (\partial\chi)^2 + (\partial\hat{\rho})^2 + 4\zeta \tilde{n} \cdot \partial\chi + 4\hat{\rho} \hat{n} \cdot \partial\chi + (\partial\hat{\theta})^2 \right. \\ \left. + (\tilde{n} \cdot \tilde{n} - \hat{n} \cdot \hat{n}) \hat{\theta}^2 \right] . \quad (4.5)$$

The spectrum is more conveniently expressed in terms of κ and ν rather than in terms of \hat{n} and \tilde{n} . Similarly to the spectrum of bosonic fluctuations in AdS₅×S⁵, we find one combination χ, ζ and $\hat{\rho}$ being massless and one each with dispersion relation

$$\omega_{\pm}(n) = \sqrt{n^2 + 2\kappa^2 \pm 2\sqrt{\kappa^4 + n^2\nu^2}}, \quad (4.6)$$

where here n denotes the mode number. There is additionally one transverse mode with mass squared $2\kappa^2 - \nu^2$. For the string moving on an S^1 inside the \mathbb{P}^3 masses of the fluctuations are quite straightforward with one longitudinal massless degree of freedom, four with mass squared $\frac{\nu^2}{4}$ and one with mass squared ν^2 .⁶ Note that in the absence of an angular momentum on \mathbb{P}^3 , the spectrum exhibits the SO(6) \simeq SU(4) symmetry of \mathbb{P}^3 . For $J \propto \nu \neq 0$ this symmetry is broken to SO(4).

As is the case for the AdS₅×S⁵ string, two of the massless modes cancel against the contribution of the diffeomorphism ghosts that arise from fixing conformal gauge. For a string spinning entirely in AdS₄ we take ν to zero and in this case the bosonic spectrum is particularly simple: we get one massive excitation with $m^2 = 4\kappa^2$, one with $m^2 = 2\kappa^2$ and six massless modes so that $\sum_{\text{bosons}} m^2 = 6\kappa^2$. As discussed in [49] we can consider the fluctuations as the Goldstone bosons (or fermions for the fermionic fluctuations to be discussed in the next section). Thus we expect the six massless modes from the \mathbb{P}^3 to remain massless to all orders in worldsheet perturbation theory.

⁵There is some ambiguity in exactly what we use as the expansion parameter however we fix this by demanding that for the analogous expansion about the BMN string the energy of a single massive excitation is $E - J = 1 + \mathcal{O}(\lambda)$.

⁶It should be mentioned that, due to the numerical factor in the second term on the right hand side of the equation (3.1), the normalization of the quadratic term of the \mathbb{P}^3 fluctuations is non-standard. While this is irrelevant at one-loop order, it must be carefully accounted for in higher-loop calculations.

4.2 Fermionic action to quadratic order

We now turn to the construction of the spectrum of fermionic fluctuations. As mentioned previously, the complete κ -gauge-invariant Green-Schwarz action on $\text{AdS}_4 \times \mathbb{P}^3$ is not known. Recently, however, Green-Schwarz [20, 21] and pure spinor [22] models based on the coset $OSp(6|4)/SU(3) \times U(1) \times SO(1,3)$ have been constructed. The resulting sigma model possesses twenty-four fermionic degrees of freedom and may be interpreted as a partial κ -gauge fixing of an action with thirty-two fermionic degrees of freedom. The remaining κ -symmetry generically removes eight of the fermions. For strings moving entirely in AdS_4 , such as the spinning folded string, a larger number of degrees of freedom becomes unphysical; the remaining κ -symmetry is enhanced and becomes capable of removing twelve fermionic degrees of freedom, instead of eight [20].

Such a small number of physical fermionic degrees of freedom is not allowed by the usual rules for the Green-Schwarz string; one would therefore expect that it is possible to use the supercoset models for the generalized spinning solutions with $J \neq 0$ but not for $J = 0$. Such a conclusion is, however, somewhat puzzling as we expect the energy to be a smooth function of J . This motivates, in part, our consideration of the generalized solutions where we can analyze the $J \rightarrow 0$ behavior and, separately, the $J = 0$ solution.⁷

For our purposes we fortunately need only the the quadratic-in-fermions part of the gauge-invariant Green-Schwarz action and this is well known to have a standard expression in terms of the target space covariant derivative:

$$L_{2F} = i(\eta^{ab}\delta^{IJ} - \epsilon^{ab}s^{IJ})\bar{\theta}^I\phi_a D_b^{JK}\theta^K \quad (4.7)$$

where $s = \text{diag}(1, -1)$ and e_a^A is the pullback of the vielbein

$$e_a^A = \partial_a X^M E_M^A \quad (4.8)$$

(here X^M denote generic target space coordinates). In type IIB theory in the presence of a 5-form flux this expression was analyzed in [15] and brought to a form resembling a two-dimensional fermionic action.

We will analyze here the type IIA string theory, with additional restrictions on the form of D_a^{JK} due to the fluxes present in the background (2.1). The structure of the action bears certain similarities with that in the type IIB theory due to the fact that the background RR fluxes are constant on the tangent space. Here however, the two fermions θ^1 and θ^2 have opposite chiralities. Defining $\mathcal{H}_{(n)} = \frac{1}{n!}\Gamma^{N_1 N_2 \dots N_n} F_{N_1 N_2 \dots N_n}$ the covariant derivative is

$$D_a^{JK} = \left(\partial_a + \frac{1}{4}\partial_a X^M \omega_M^{AB} \Gamma_{AB} \right) \delta^{JK} - \frac{1}{8}\partial_a X^M E_M^A H_{ABC} \Gamma^{BC} (\sigma_3)^{JK} + \frac{1}{8}e^\phi [F_{(0)}(\sigma_1)^{JK} + \mathcal{H}_{(2)}(i\sigma_2)^{JK} + \mathcal{H}'_{(4)}(\sigma_1)^{JK}] \phi_a \quad (4.9)$$

with σ_i being the Pauli matrices and the modified form field strength F'_4 given, as usual, by

$$F'_4 = F_4 - H \wedge C_1 \quad (4.10)$$

⁷After this work appeared a similar calculation using the coset approach [50] was submitted which found that the $J \rightarrow 0$ limit was smooth and in agreement with our calculation.

In the coordinates (2.2) the spin connection reads:

$$\omega^{01} = -\omega^{10} = \sinh \rho dt \qquad \omega^{21} = -\omega^{12} = \cosh \rho d\theta \qquad (4.11)$$

$$\omega^{31} = -\omega^{13} = \cosh \rho \sin \theta d\phi \qquad \omega^{32} = -\omega^{23} = \cos \theta d\phi. \qquad (4.12)$$

With regard to the spin connection for the compact \mathbb{P}^3 we note that using local Lorentz transformations it is always possible to choose the spin connection to vanish along a chosen direction — in particular φ_3 . It turns out that the coordinates (2.3) together with the choice of φ_3 mentioned above realize this observation. Thus, for spinning string solutions carrying a single charge in the space transverse to AdS, the explicit form of the spin connection is not necessary for the calculation of the spectrum of quadratic fluctuations. If the profile in the transverse space involves a single (isometric) field, then one also does not — for the same purpose — need to make sure that the full metric is written in the coordinates adapted to the vanishing spin connection. Indeed, the spectrum is invariant under coordinate transformations, so one can compute the bosonic spectrum in any suitable coordinate system.

4.2.1 The $(S, J = 0)$ string

Let us consider first the solution with vanishing angular momentum in the transverse space. A reason for analyzing this configuration separately (rather than as a limit of $J \neq 0$ configurations which will be discussed later) is to test explicitly the continuity of the energy and of the natural κ -gauge condition as a function of J . Moreover, the details of the calculation compared to those for the $J \neq 0$ configurations may point the origin of the enhancement of the κ symmetry of the $OSp(6|4)$ models. As was exploited extensively in the calculation of one-loop corrections to the energy of classical strings in $AdS_5 \times S^5$, no bosonic fluctuations appear in the quadratic fermion action; one simply evaluates (4.7) on the classical solution. Using the fact that from (3.9) we have $\hat{n} = (\kappa, 0)$, $\tilde{n} = (0, \kappa)$ and $\hat{m} = (0, 0)$ it follows that

$$\not\epsilon_a = \frac{R_{\text{string}}}{2} [\hat{n}_a (\cosh \bar{\rho} \Gamma_0 + \sinh \bar{\rho} \Gamma_3) + \tilde{n}_a \Gamma_1] \quad . \qquad (4.13)$$

Also, the spin connection evaluated on the background solution is:

$$\partial_a X^M \omega_M^{AB} \Gamma_{AB} = 2\hat{n}_a (\sinh \bar{\rho} \Gamma_0 + \cosh \bar{\rho} \Gamma_3) \Gamma_1 \qquad (4.14)$$

The $\bar{\rho}$ dependence may be removed by a rotation (boost) in the (03) plane:

$$\begin{aligned} S &= \cosh \frac{\bar{\rho}}{2} + \sinh \frac{\bar{\rho}}{2} \Gamma_{03} \qquad (4.15) \\ (\cosh \bar{\rho} \Gamma_0 + \sinh \bar{\rho} \Gamma_3) &= S \Gamma_0 S^{-1} \\ (\sinh \bar{\rho} \Gamma_0 + \cosh \bar{\rho} \Gamma_3) &= S \Gamma_3 S^{-1} \quad . \end{aligned}$$

This is absorbed by a field redefinition of the fermions

$$\theta^I = S \psi^I \qquad (4.16)$$

which in turn introduces an additional connection component:

$$S^{-1}\partial_a S = \frac{1}{2}\tilde{n}_a\Gamma_{03} \quad (4.17)$$

Thus, we need to expand:

$$\begin{aligned} \mathcal{L}_{ab}^{IJ} &= \bar{\theta}^I \not{\epsilon}_a \left(\partial_b + \frac{1}{4}\omega_b^{AB}\Gamma_{AB} \right) \theta^J + \frac{1}{8}e^\phi \bar{\theta}^I \not{\epsilon}_a \left[\mathcal{H}_{(2)}(i\sigma_2)^{JK} + \mathcal{H}_{(4)}(\sigma_1)^{JK} \right] \not{\epsilon}_b \theta^K \\ &= \frac{R_{\text{string}}}{2} \left[\bar{\psi}^I (\hat{n}_a\Gamma_0 + \tilde{n}_a\Gamma_1) \left(\partial_b + \frac{1}{2}(\tilde{n}_b\Gamma_0 - \hat{n}_b\Gamma_1)\Gamma_3 \right) \psi^J \right. \\ &\quad \left. + \frac{R_{\text{string}}}{16} e^\phi \bar{\psi}^I \left[\mathcal{H}_{(2)}(i\sigma_2)^{JK} - \mathcal{H}_{(4)}(\sigma_1)^{JK} \right] (\hat{n}_a\Gamma_0 + \tilde{n}_a\Gamma_1)(\hat{n}_b\Gamma_0 + \tilde{n}_b\Gamma_1) \psi^K \right]. \end{aligned} \quad (4.18)$$

In the flux term we used the fact that F_2 does not have components in the AdS direction so it commutes with Γ_0 and Γ_1 while $\mathcal{H}_{(4)} \propto \Gamma_{0123}$ so it anticommutes with $(\hat{n}_a\Gamma_0 + \tilde{n}_a\Gamma_1)$. In the second term in the parenthesis, all factors of R and k cancel out once the expressions of the dilaton and forms are included.

Using the fact that $(\eta^{ab}\delta^{IJ} - \epsilon^{ab}{}_s{}^{IJ})$ is diagonal in the indices I, J it is possible to simplify somewhat the first term above, which we will denote by \mathcal{D}_{ab}^{IJ} . Indeed, opening the parenthesis,

$$\begin{aligned} \mathcal{D}_{ab}^{IJ} &= \bar{\psi}^I (\hat{n}_a\Gamma_0 + \tilde{n}_a\Gamma_1) \left(\partial_b + \frac{1}{2}(\tilde{n}_b\Gamma_0 - \hat{n}_b\Gamma_1)\Gamma_3 \right) \psi^J \\ &= \bar{\psi}^I (\hat{n}_a\Gamma_0 + \tilde{n}_a\Gamma_1) \partial_b \psi^J - \frac{1}{2}(\hat{n}_a\hat{n}_b + \tilde{n}_a\tilde{n}_b) \bar{\psi}^I \Gamma_{013} \psi^J + \mathcal{O}(\bar{\psi}^I \Gamma_3 \psi^J) \end{aligned} \quad (4.19)$$

it is not hard to identify terms which vanish, if $I = J$, due to the chirality of fermions.

The two terms arising in the sum the indices I, J in $(\eta^{ab}\delta^{IJ} - \epsilon^{ab}{}_s{}^{IJ})\mathcal{D}_{ab}^{IJ}$ are both of the same type:

$$(\eta^{ab} + \eta\epsilon^{ab})\mathcal{D}_{ab}^{II} = -\bar{\psi}^I \Gamma_0 (1 - \eta\Gamma_0\Gamma_1) \partial_0 \psi^I + \bar{\psi}^I \Gamma_1 (1 - \eta\Gamma_0\Gamma_1) \partial_1 \psi^I \quad (4.20)$$

where we used the Virasoro constraint $\hat{n} \cdot \hat{n} + \tilde{n} \cdot \tilde{n} = 0$. Here $\eta = -1$ if $I = 1$ and $\eta = +1$ if $I = 2$. It is useful to note the explicit appearance of projection operators

$$\mathcal{P}_\pm = \frac{1}{2}(1 \pm \Gamma_{01}) \quad ; \quad (4.21)$$

this is a consequence of the κ -symmetry of the action.

The trivial multiplication of vielbeine $\not{\epsilon}_a \not{\epsilon}_b = (\hat{n}_a\Gamma_0 + \tilde{n}_a\Gamma_1)(\hat{n}_b\Gamma_0 + \tilde{n}_b\Gamma_1)$ leads to a simple expression for the vielbein-dependent factor in the flux-dependent term in (4.18). It is again a sum of two terms of the type

$$(\eta^{ab} + \eta\epsilon^{ab})\not{\epsilon}_a \not{\epsilon}_b = 2(1 + \eta\Gamma_{01}) \quad (4.22)$$

where we made use of the explicit expressions of the vectors n and \tilde{n} to write $\epsilon^{ab}\hat{n}_a\tilde{n}_b = 1$ and $-\hat{n} \cdot \hat{n} + \tilde{n} \cdot \tilde{n} = +2$ and, as before, $\eta = -1$ for $I = 1$ and $\eta = +1$ for $I = 2$. Note again the appearance of the projectors \mathcal{P}_η .

The action is easy to construct by starting from (4.7); skipping trivial details, the result is

$$\begin{aligned}
\frac{2}{iR_{\text{string}}}L_{2F} &= \frac{2}{R_{\text{string}}}(\eta^{ab}\delta^{IJ} - s^{IJ}\epsilon^{ab})\mathcal{L}_{ab}^{IJ} \\
&= -\bar{\psi}^1\Gamma_0(1 + \Gamma_0\Gamma_1)\partial_0\psi^1 + \bar{\psi}^1\Gamma_1(1 + \Gamma_0\Gamma_1)\partial_1\psi^1 \\
&\quad -\bar{\psi}^2\Gamma_0(1 - \Gamma_0\Gamma_1)\partial_0\psi^2 + \bar{\psi}^2\Gamma_1(1 - \Gamma_0\Gamma_1)\partial_1\psi^2 \\
&\quad + \frac{R_{\text{string}}}{8}e^\phi\bar{\psi}^1 [\mathcal{H}_{(2)}(+1) - \mathcal{H}_{(4)}(+1)] (1 - \Gamma_{01})\psi^2 \\
&\quad + \frac{R_{\text{string}}}{8}e^\phi\bar{\psi}^2 [\mathcal{H}_{(2)}(-1) - \mathcal{H}_{(4)}(+1)] (1 + \Gamma_{01})\psi^1 . \quad (4.23)
\end{aligned}$$

At this stage it is useful to recall that ψ^1 and ψ^2 are spinors of opposite chirality — with Γ_{-1} the ten-dimensional chirality operator, $\Gamma_{-1}\psi^1 = \psi^1$ and $\Gamma_{-1}\psi^2 = -\psi^2$ — and thus may be assembled into a single, non-chiral ten-dimensional spinor $\psi = \psi^1 + \psi^2$. In terms of this new field the action takes a very simple form:

$$L_{2F} = \frac{iR_{\text{string}}}{2} \left(2\bar{\psi}(-\Gamma^0\partial_0 + \Gamma^1\partial_1)\mathcal{P}_+\psi - \frac{R_{\text{string}}}{4}e^\phi\bar{\psi} [\mathcal{H}_{(2)}\Gamma_{-1} + \mathcal{H}_{(4)}] \mathcal{P}_+\psi \right) . \quad (4.24)$$

This action is still invariant under local κ -transformations, a fact reflected by the manifest appearance of a projector \mathcal{P}_+ in all terms in the action. It is only natural to choose the gauge

$$\mathcal{P}_+\psi = \psi , \quad (4.25)$$

which eliminates from the fermion fields the components not appearing in the Lagrangian. This algebraic gauge, which is similar to the light-cone gauge, introduces no κ -symmetry ghosts.

For explicit calculations it is necessary to expand also the last term in the action (4.24) using the explicit form of the form fields; the relative factor of R_{string} with the derivative term cancels out and we find

$$-\frac{R_{\text{string}}}{4}e^\phi\bar{\psi} [\mathcal{H}_{(2)}\Gamma_{-1} + \mathcal{H}_{(4)}] \psi = -\frac{1}{4} [+2(\Gamma_{45} - \Gamma_{67} + \Gamma_{89})\Gamma_{-1} + 6\Gamma_{0123}] . \quad (4.26)$$

The spectrum of fermion quadratic operator (4.24) may be found by evaluating its eigenvalues and setting them to zero. It turns out that there are two massless and six massive modes with unit mass:

$$\omega_{1,2}(n) = |n| \quad \omega_{3,4,5,6,7,8}(n) = \sqrt{n^2 + \kappa^2} . \quad (4.27)$$

Note that, similarly to the bosonic spectrum, $\sum_{i=1}^8 m_i^2 = 6\kappa^2$; therefore, the one-loop correction to the energy of the $(S, J = 0)$ string is finite. We will evaluate it in section (5).

The structure of this spectrum could have been anticipated from symmetry considerations. Indeed, as reviewed in the introduction, the supersymmetries form a $\mathbf{6}_0 \oplus \mathbf{1}_2 \oplus \mathbf{1}_{-2}$ representation of the global symmetry group $SU(4) \times U(1)$. Thus, we should expect six modes of equal masses. An additional \mathbb{Z}_2 (charge conjugation) symmetry changing the sign of the $U(1)$ charges suggests that the remaining two modes should also have equal masses.

4.2.2 The $(S, J \neq 0)$ string

The inclusion of a single angular momentum on \mathbb{P}^3 is technically quite straightforward. The main difference is that now all three vectors (3.9) are nontrivial and given by $\hat{n} = (\kappa, 0)$, $\tilde{n} = (0, \mu)$ and $\hat{m} = (\nu, 0)$.

Since the angular momentum on \mathbb{P}^3 is described by a linear profile along an isometry direction, it introduces no additional worldsheet coordinate dependence in the fermion action besides the one due to the AdS₄ part of the solution. As for $J = 0$ this latter dependence may be eliminated by the rotation (4.15). After this rotation, the vielbein and the spin connection modified to include the effects of the rotation (4.17) are:

$$\begin{aligned} \not{\epsilon}_a &= \frac{R_{\text{string}}}{2} [\hat{n}_a \Gamma_0 + \tilde{n}_a \Gamma_1 + \hat{m}_a \Gamma_9] \\ \frac{1}{4} \tilde{\omega}_a^{AB} \Gamma_{AB} &= \frac{1}{2} (\tilde{n}_a \Gamma_0 - \hat{n}_a \Gamma_1) \Gamma_3 \end{aligned} \quad (4.28)$$

$$\frac{1}{4} \not{\epsilon}_a \tilde{\omega}_b^{AB} \Gamma_{AB} = \frac{R_{\text{string}}}{2} \left[-\frac{1}{2} (\hat{n}_a \hat{n}_b + \tilde{n}_a \tilde{n}_b) \Gamma_{013} + \frac{1}{2} \hat{m}_a \tilde{n}_b \Gamma_{039} - \frac{1}{2} \hat{m}_a n_b \Gamma_{139} \right]. \quad (4.29)$$

The two terms arising from the gravitational covariant derivative continue to have a similar structure, up to some signs (denoted by η) which again related to the chirality of the spinors:

$$\begin{aligned} \frac{2}{R_{\text{string}}} (\eta^{ab} + \eta \epsilon^{ab}) \mathcal{D}_{ab}^{II} &= (\eta^{ab} + \eta \epsilon^{ab}) \bar{\psi}^I (\hat{n}_a \Gamma_0 + \tilde{n}_a \Gamma_1 + \hat{m}_a \Gamma_9) \partial_b \psi^I \\ &\quad - \frac{1}{2} (\hat{n} \cdot \hat{n} + \tilde{n} \cdot \tilde{n}) \bar{\psi}^I \Gamma_{013} \psi^I \\ &\quad - \frac{1}{2} \hat{m} \cdot \hat{n} \bar{\psi}^I \Gamma_{139} \psi^I + \frac{\eta}{2} \hat{m} \times \tilde{n} \bar{\psi}^I \Gamma_{039} \psi^I \\ &= -\bar{\psi}^I (\kappa \Gamma_0 + \eta \mu \Gamma_1 + \nu \Gamma_9) \partial_0 \psi^I + \bar{\psi}^I (\eta \kappa \Gamma_0 + \mu \Gamma_1 + \eta \nu \Gamma_9) \partial_1 \psi^I \\ &\quad + \frac{1}{2} \nu^2 \bar{\psi}^I \Gamma_{013} \psi^I + \frac{1}{2} \kappa \nu \bar{\psi}^I \Gamma_{139} \psi^I + \frac{\eta}{2} \nu \mu \bar{\psi}^I \Gamma_{039} \psi^I. \end{aligned} \quad (4.30)$$

It is easy to identify in the derivative terms a projector ($\mathcal{P}_\eta^2 = \mathcal{P}_\eta$) analogous to the one in equation (4.21); it is:

$$\mathcal{P}_\eta = \frac{1}{2} \left(1 + \eta \left(\frac{\kappa}{\mu} \Gamma_0 + \frac{\nu}{\mu} \Gamma_9 \right) \Gamma_1 \right) \quad \eta = \pm. \quad (4.32)$$

Using it and introducing the same unconstrained, non-chiral ten-dimensional spinor as before $\psi = \psi^1 + \psi^2$ the equation (4.30) can be reorganized as:

$$\begin{aligned} (\eta^{ab} \delta^{IJ} - \epsilon^{ab} s^{IJ}) \mathcal{D}_{ab}^{IJ} &= \frac{R_{\text{string}}}{2} \bar{\psi} \left[-2(\kappa \Gamma_0 + \nu \Gamma_9) \mathcal{P}_+ \partial_0 + 2\mu \Gamma_1 \mathcal{P}_+ \partial_1 \right. \\ &\quad \left. + \frac{1}{2} \nu (\nu \Gamma_0 + \kappa \Gamma_9) \Gamma_{13} - \frac{1}{2} \nu \mu \Gamma_{039} \right] \psi. \end{aligned} \quad (4.33)$$

Note that, unlike the string spinning only in AdS, there is a nontrivial connection term; these terms vanish as $\nu \sim J \rightarrow 0$ and the derivative terms reduce to those of the previous section.

To simplify the flux contribution it is useful to use the explicit forms of the vectors n , \tilde{n} and m and to split the 2-form into a part depending on the \mathbb{P}^3 isometry direction (i.e. Γ_9), $H_2^{(1)}$, and the rest, $H_2^{(2)}$:

$$H_2 = H_2^{(1)} + H_2^{(2)} . \quad (4.34)$$

In terms of these components, the flux terms are:

$$\begin{aligned} & \left(\frac{2}{R_{\text{string}}} \right)^2 e^\phi (\eta^{ab} \delta^{IJ} - \epsilon^{ab} \epsilon^{IJ}) \bar{\psi}^I \not{\epsilon}_a [H_{(2)}(i\sigma_2)^{JK} + H_{(4)}(\sigma_1)^{JK}] \not{\epsilon}_b \psi^K \\ &= \bar{\psi} H_2^{(2)} \Gamma_{-1} (2\mu^2 + 2\kappa\mu\Gamma_{01}\Gamma_{-1} - 2\mu\nu\Gamma_{19}\Gamma_{-1}) \psi \\ & \quad + \bar{\psi} (H_2^{(2)} \Gamma_{-1} - H_4) (2\kappa^2 - 2\kappa\nu\Gamma_{09} + 2\kappa\mu\Gamma_{01}\Gamma_{-1}) \psi \end{aligned} \quad (4.35)$$

It is not hard to expose the projectors in this expression; restoring the numerical coefficient of the flux term in the covariant derivative and making use of the explicit expressions for the form fields we find that the contribution of the form fields to the fermion action to quadratic order in fermions and to leading order in the expansion in bosonic fluctuations is

$$\begin{aligned} & \frac{1}{8} e^\phi (\eta^{ab} \delta^{IJ} - \epsilon^{ab} \epsilon^{IJ}) \bar{\psi}^I \not{\epsilon}_a [H_{(2)}(i\sigma_2)^{JK} + H_{(4)}(\sigma_1)^{JK}] \not{\epsilon}_b \psi^K \\ &= \frac{R_{\text{string}}}{2} \left[\frac{1}{16} (4\mu^2) \bar{\psi} (-\Gamma_{57} + \Gamma_{68}) \Gamma_{-1} \mathcal{P}_+ \psi + \frac{1}{16} (4\mu\kappa) \bar{\psi} (-\Gamma_{49}\Gamma_{-1} + 3\Gamma_{0123}) \mathcal{P}_+ \psi \right]. \end{aligned} \quad (4.36)$$

Combining the derivative (4.33) and the flux terms (4.36) it is easy to find the relevant gauge-invariant Lagrangian (a constant rotation in the (09) plane may be used to slightly simplify the derivative term):

$$\begin{aligned} \frac{2}{iR_{\text{string}}} \mathcal{L}_{2F} &= \bar{\psi} \left[-2(\kappa\Gamma_0 + \nu\Gamma_9) \mathcal{P}_+ \partial_0 + 2\mu\Gamma_1 \mathcal{P}_+ \partial_1 + \frac{1}{2}\nu(\nu\Gamma_0 + \kappa\Gamma_9) \Gamma_{13} - \frac{1}{2}\kappa\mu\Gamma_{039} \right] \psi \\ & \quad - \frac{1}{16} [(4\mu^2) \bar{\psi} (-2\Gamma_{57} + 2\Gamma_{68}) \Gamma_{-1} \mathcal{P}_+ \psi + (4\mu\kappa) \bar{\psi} (-2\Gamma_{49}\Gamma_{-1} + 6\Gamma_{0123}) \mathcal{P}_+ \psi]. \end{aligned} \quad (4.37)$$

As before, the manifest appearance of \mathcal{P}_+ suggests that a natural gauge condition is

$$\mathcal{P}_+ \psi = \psi , \quad (4.38)$$

in analogy to the $J = 0$ analysis. As in that case, this gauge condition does not introduce any κ -symmetry ghosts. It is moreover easy to see that the limit $\nu \rightarrow 0$ quickly leads to the equation (4.24), implying that the gauge condition is a smooth function of J .

The energy spectrum of quadratic fluctuations can be found by first setting to zero the eigenvalues of the quadratic fluctuations operator; the result, which may be checked by a variety of means, is that

$$\begin{aligned} \omega_{1,2,\pm}(n) &= \pm \frac{\nu}{2} + \sqrt{n^2 + \kappa^2} \\ \omega_{3,4,\pm}(n) &= \frac{1}{\sqrt{2}} \sqrt{\kappa^2 + 2n^2 \pm \sqrt{\kappa^4 + 4\nu^2 n^2}} \end{aligned} \quad (4.39)$$

Thus, we find four modes with unit mass and the other four modes have more complicated dispersion relations which are similar to those for some of the bosonic AdS fluctuations (4.6).

It is interesting to note that the massless fermion modes present at $J = 0$ are now lifted. The fact that four modes continue to have equal masses (up to a time-dependent rotation of their wave functions) is consistent with (and in fact should be expected from) the fact that the worldsheet background breaks the symmetry of \mathbb{P}^3 from $SO(6)$ to $SO(4)$.

5. One-loop correction to string energies

Given the spectrum of fluctuations we found in previous sections, the one-loop correction to the string energy may be computed in a variety of ways. An important subtlety is that the relation between the parameters of the solution and the field theory charges may receive quantum corrections. Such effects may be captured either in the Hamiltonian formalism [15] or in the Lagrangian formalism [35]. In the latter approach the fundamental quantity is the worldsheet partition function in the presence of chemical potentials for all charges. The target space energy is found by Legendre-transforming the logarithm of the partition function with respect to the chemical potentials. In the process one also uses the quantum Virasoro constraint, which sets to zero the quantum expectation value of the worldsheet Hamiltonian.

The results obtained through these two methods imply that such modifications to the relation between charges and parameters of the classical solution are irrelevant in a one-loop calculation. It is perhaps more convenient to use the expression for the string energy in conformal gauge in terms of the fluctuation fields derived in appendix A of [15]:

$$E_1 = \frac{1}{\kappa} \langle \Psi | H_2 | \Psi \rangle \tag{5.1}$$

with $H_2 = \int \frac{d\sigma}{2\pi} \mathcal{H}_2(\tilde{t}, \tilde{\phi}, \dots)$ being the quadratic worldsheet Hamiltonian corresponding the fluctuation action at this order. For the spinning string the classical solution spontaneously breaks supersymmetry and we expect to find a non-trivial correction at one-loop. We begin with the simpler ($S, J = 0$) case and then proceed to the general solution.

5.1 ($S, J = 0$)

For the case ($S, J = 0$) we have in the scaling limit that the energy is given by the sum over frequencies

$$E_1 = \frac{1}{2\kappa} \sum_{n=-\infty}^{\infty} K_n + \mathcal{O}(\kappa^0) \tag{5.2}$$

with $K_n = \sqrt{n^2 + 4\kappa^2} + \sqrt{n^2 + 2\kappa^2} + 6\sqrt{n^2} - 6\sqrt{n^2 + \kappa^2} - 2\sqrt{n^2}$. It is worth noting that while the series in the equation above is absolutely convergent, the partial sums over individual frequencies are divergent. The organization of frequencies shown in (5.2) is dictated by the form of the Hamiltonian derived from the worldsheet theory. Moreover, the same analysis implies that, as is usual for field theories, any regularization should be carried out at the

level of the action rather than in a particular frequency sum. While without a worldsheet justification, one may nonetheless make different choices for combining the individual terms in the infinite sum and this can give rise to finite differences (for a discussion of this point in the current context see [51, 52]). The $\mathcal{O}(\kappa^0)$ terms become subleading in the scaling limit, $\kappa \gg 1$, and further, the sum can be replaced by an integral. After rescaling the worldsheet mode numbers, n , and introducing the continuous worldsheet momentum, p , we have

$$E_1 = \kappa \int_0^\infty dp \sqrt{p^2 + 4} + \sqrt{p^2 + 2} + 6\sqrt{p^2} - 6\sqrt{p^2 + 1} - 2\sqrt{p^2} + \mathcal{O}(\kappa^0). \quad (5.3)$$

It is straightforward to evaluate this integral by imposing a cutoff, performing the individual integrals and taking the cutoff to infinity. Expanding at large values of the cutoff one can check that the quadratic and logarithmic UV divergences vanish. The leading finite piece is given by

$$\begin{aligned} E_1 &= -\kappa \frac{5}{2} \ln 2 + \mathcal{O}(\kappa^0) \\ &= -\frac{5 \ln 2}{2\pi} \ln S + \mathcal{O}(\ln^0 S) \quad . \end{aligned} \quad (5.4)$$

Thus we see that, as for the AdS₅ case, the one-loop piece continues to scale as $\ln S$ and there is no stronger $\ln^\alpha S$, $\alpha > 1$, dependence. In fact we expect, not least on simple dimensional grounds, that this structure will continue to all orders at strong coupling and can be interpolated to match the weak coupling result.

5.2 ($S, J \neq 0$)

We can now use essentially the same method for the generalized (S, J) string solution with two non-vanishing charges. In this case the sum of frequencies of the bosonic fluctuations, (4.6) and below, and the fermionic fluctuations, (4.39), is

$$\begin{aligned} K_n &= \sqrt{n^2 + 2\kappa^2 + 2\sqrt{\kappa^4 + n^2\nu^2}} + \sqrt{n^2 + 2\kappa^2 - 2\sqrt{\kappa^4 + n^2\nu^2}} \\ &\quad + \sqrt{n^2 + 2\kappa^2 - \nu^2} + 4\sqrt{n^2 + \frac{\nu^2}{4}} + \sqrt{n^2 + \nu^2} - 4\sqrt{n^2 + \kappa^2} \\ &\quad - 2 \left(\frac{1}{\sqrt{2}} \sqrt{2n^2 + \kappa^2 + \sqrt{\kappa^4 + 4n^2\nu^2}} + \frac{1}{\sqrt{2}} \sqrt{2n^2 + \kappa^2 - \sqrt{\kappa^4 + 4n^2\nu^2}} \right). \end{aligned} \quad (5.5)$$

We again replace the discrete sum over mode numbers by an integral which, with the help of identities and changes of variables from appendix A, results in a one-loop correction to the energy of

$$\begin{aligned} E_1 &= \frac{\nu}{2u} \left[- (1 - u^2) + \sqrt{1 - u^2} - 2u^2 \ln u \right. \\ &\quad \left. - (2 - u^2) \ln \left(\sqrt{2 - u^2} (1 + \sqrt{1 - u^2}) \right) - 2(1 - u^2) \ln 2 \right] \quad . \end{aligned} \quad (5.6)$$

which is seen to be remarkably similar to the AdS₅ × S⁵ result though with some modifications. Here we have used the parameter

$$u = \frac{\nu}{\kappa} = \frac{\ell}{\sqrt{1 + \ell^2}}, \quad \ell = \frac{\nu}{\mu} = \frac{\mathcal{J}}{2 \ln S} \quad (5.7)$$

and it is straightforward to see that in the $u \rightarrow 0$ limit it reduces to equation (5.4). This generalized scaling function is a useful tool in studying the AdS₅/CFT₄ duality and it is to be expected that it will also be so in the case at hand.

6. Comparison with the Bethe ansatz and outlook

The dilatation operator of the $\mathcal{N} = 6$ Chern-Simons theory was shown [16, 18], to leading order in the scalar sector, to be equivalent to the Hamiltonian of an integrable (alternating) spin chain. It was moreover argued that the worldsheet theory in the dual supergravity background is also classically integrable [20, 21]. It is tempting to infer that integrability potentially exists for finite values of the 't Hooft coupling as well. This conjecture is based on the nontrivial assumption that the anomaly of the conservation of the hidden charges present in the bosonic \mathbb{P}^3 sigma model is canceled in the full Green-Schwarz theory. It would be important to have direct tests of this assumption.

With this starting point, and using the observation that the transformation rules of the spin chain excitations are similar to those of the spin chain excitations in $\mathcal{N} = 4$ SYM, all-order Bethe equations have been conjectured [23] for the $\mathcal{N} = 6$ Chern-Simons theory. As in $\mathcal{N} = 4$ SYM, the tensor structure of the relevant scattering matrices is fixed by symmetries. The difference compared to the four-dimensional case is that the magnon dispersion relation acquires an overall numerical factor and in both the magnon dispersion relation and the S-matrix the 't Hooft coupling enters through an arbitrary function $h(\lambda)$ ⁸

$$\epsilon(p) = \frac{1}{2} \sqrt{1 + 16h(\lambda)^2 \sin^2 \frac{p}{2}} . \quad (6.1)$$

In $\mathcal{N} = 4$ SYM one has $h(\lambda) = \sqrt{\lambda}/4\pi$ while in the $\mathcal{N} = 6$ Chern-Simons theory

$$h(\lambda) = \begin{cases} \lambda + \mathcal{O}(\lambda^3) & \lambda \ll 1 \\ \sqrt{\frac{\lambda}{2}} + \mathcal{O}(1) & \lambda \gg 1 \end{cases} . \quad (6.2)$$

It was further argued that, up to the same function $h(\lambda)$, the dressing phase is the same as that of the scattering matrix of the $\mathcal{N} = 4$ SYM spin chain.

This relation between scattering matrices and dispersion relations implies in turn that most anomalous dimensions in the $\mathcal{N} = 6$ Chern-Simons theory enjoy simple relations with those of $\mathcal{N} = 4$ SYM theory. For example, it was argued in [23] that for the universal scaling functions this relation is

$$f_{\text{CS}}(\lambda) = \frac{1}{2} f_{\mathcal{N}=4}(\lambda) \Big|_{\sqrt{\lambda} \rightarrow 4\pi h(\lambda)} . \quad (6.3)$$

Using the result from the algebraic curve calculation [53] that the constant term in $h(\lambda)$ vanishes in the regularization scheme adapted to the algebraic curve calculation⁹ and of

⁸This function may be fixed by a direct calculation of the magnon dispersion relation in the $\mathcal{N} = 6$ Chern-Simons theory.

⁹Different regularization schemes, such as one more natural from the worldsheet perspective, can give rise to finite differences which in turn can be interpreted, in part, as an $\mathcal{O}(1)$ corrections to the function $h(\lambda)$; this possibility was discussed at length in [51, 52] which appeared after this paper. An important unsolved question is the consistency of the various regularization schemes with integrability and the ABA.

the known strong coupling expansion of the universal scaling function,

$$f_{\mathcal{N}=4}(\lambda) = \frac{1}{\pi} \left(\sqrt{\lambda} - 3 \ln 2 + \mathcal{O} \left(\frac{1}{\sqrt{\lambda}} \right) \right) , \quad (6.4)$$

it is easy to find that

$$f_{CS}(\lambda) = \frac{1}{2} f_{\mathcal{N}=4}(\lambda) \Big|_{\sqrt{\lambda} \rightarrow 4\pi h(\lambda)} = \sqrt{2\lambda} - \frac{3 \ln 2}{2\pi} + \mathcal{O} \left(\frac{1}{\sqrt{\lambda}} \right) \quad (6.5)$$

The first term matches (by construction) the leading order in the strong coupling expansion of the spinning folded string energy (3.14). The second term above however departs from the worldsheet predictions (5.4) for the next-to-leading order correction to the universal scaling function.

In the same spirit one may compare (5.6) with the consequence of the conjectured Bethe ansatz for the $\mathcal{N} = 6$ Chern-Simons theory. Instead of¹⁰

$$f_{CS} \left(\lambda, \frac{J}{\ln S} \right) = \frac{1}{2} f_{\mathcal{N}=4} \left(\lambda, \frac{J}{\ln S} \right) \Big|_{\sqrt{\lambda} \rightarrow 4\pi h(\lambda)} , \quad (6.6)$$

it is easy to see that the leading and next-to-leading terms in the string coupling expansion of the generalized scaling function $f_{CS}(\lambda, \ell)$ are consistent with

$$f_{CS} \left(\lambda, \frac{\mathcal{J}}{\ln S} \right) = \frac{1}{2} f_{\mathcal{N}=4} \left(\lambda, \frac{\mathcal{J}}{\ln S} \right) \Big|_{\sqrt{\lambda} \rightarrow 4\pi h(\lambda)} - \frac{\nu}{u} (1 - u^2) \ln 2 \quad (6.7)$$

where $f_{\mathcal{N}=4}(\lambda, \frac{\mathcal{J}}{\ln S})$ is given in [34, 15].

Though the resolution of this puzzle is not immediately apparent, several possibilities present themselves. For example, it may be possible that twist-two operators dual to the spinning folded string have been misidentified. It may also be possible that the problem lies either with the assumption that integrability survives beyond the leading order in the strong coupling expansion or with the precise expression for the scattering matrix. Since its tensor structure is determined by symmetries whose action is closely related to the action of symmetry generators in $\mathcal{N} = 4$ SYM, it may be that the dressing phase receives additional next-to-leading order corrections. Perhaps a profitable route to finding these corrections is to follow the strategy of [54] and construct the phase by matching it with the one-loop corrections to the circular string rotating entirely in AdS_4 . Another approach would, of course, be a direct solution of the crossing equation. The similarity of the symmetry groups of the scattering matrix of the worldsheet theory in $\text{AdS}_4 \times \mathbb{P}^3$ and $\text{AdS}_5 \times S^5$ suggests however that the correction to the dressing phase, if any, is a solution of the homogeneous crossing equation.

For the spinning string in $\text{AdS}_5 \times S^5$ it has proven possible to extend the calculation of the quantum corrections to two-loops [55, 56, 35] and it would certainly be interesting to repeat that calculation in the current context (for related discussions and some unresolved issues in the comparison of the two-loop strong coupling calculation and the ABA

¹⁰We are grateful to P. Vieira for pointing out a difference of 2 in our definition of J and that used in [23].

in $AdS_5 \times S^5$ see [57, 58]). In the absence of an argument that there exists a κ -gauge in which the action becomes quadratic in fermions, a prerequisite for a higher-loop calculation is knowledge of the contributions to the string action from terms quartic (and higher) in fermions. One could hope to possibly use the $OSp(6|4)$ coset sigma-model [20–22] though due to the enhanced κ -symmetry at $J = 0$, such a calculation appears challenging at first sight. Alternatively one can derive the type IIA string action by doubly dimensionally reducing the supermembrane action [59–61]

$$S = - \int d^3\zeta \sqrt{-\det g(Z(\zeta))} + \int_{M_3} B \tag{6.8}$$

where $Z = (X^\mu, \theta^\alpha)$ are the eleven dimensional target superspace coordinates, $\zeta = (\tau, \sigma, \sigma_3)$ are the worldvolume coordinates,

$$g_{\hat{i}\hat{j}} = \partial_{\hat{i}} Z^M \partial_{\hat{j}} Z^N E_M^{\hat{r}} E_N^{\hat{s}} \eta_{\hat{r}\hat{s}} \tag{6.9}$$

is the pullback of the supervielbein to the worldvolume and B is the pullback of the superthree-form. This procedure can be somewhat involved and has been explicitly done only to quadratic order in fermions for generic bosonic backgrounds. However for the case of $AdS_4 \times S^7/\mathbb{Z}_k$ due to the large degree of symmetry it may be possible to carry it out to higher orders starting from the supermembrane action of [62] where explicit expressions for the supervielbein and B are given to all orders in fermions.

A further appeal of such an approach relates to the exactness of the $AdS_4 \times S^7$ geometry and its consequences. As was argued by Kallosh and Rajaraman [63] the $AdS_4 \times S^7$ geometry is exact in that it cannot receive ℓ_p corrections which are consistent with supersymmetry. While the \mathbb{Z}_k orbifold relating it to $AdS_4 \times \mathbb{P}^3$ breaks some of the supersymmetry for $k > 2$, it is reasonable to expect that this geometry remains unchanged and thus that the type IIA solution $AdS_4 \times \mathbb{P}^3$ does not receive α' corrections (up to perhaps a finite renormalization of the radius of the space).

Another possible approach to extracting higher-loop information is suggested by the work of Alday and Maldacena who showed, [49], that for $AdS_5 \times S^5$ the leading logarithmic dependence on u is described by a two-dimensional $O(6)$ sigma model. At the level of the string worldsheet one may justify this by integrating out the massive modes and constructing in this way an effective action for the light modes.¹¹ Similar reasoning suggests that here the leading logarithmic dependence in u may be captured by a \mathbb{P}^3 model coupled to two light fermions - the light degrees of freedom in the current model. While it is known that the bosonic \mathbb{P}^3 model is not integrable at the quantum level due to an anomaly in the conservation of the non-local charges, [64], it is possible to couple the theory to fermions such that the anomaly cancels. Such are the minimal or the supersymmetric couplings, see for example [65]; it would be interesting to check whether the same is true in this case. One would then be able to predict the coefficients of the leading and first subleading $\ln u$ terms to all orders in the strong coupling expansion.

¹¹“Light” stands for masses of order ν or u .

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A. Integrals

The sum of frequencies of bosonic and fermionic fluctuations can be put in a form which can be integrated without difficulty using the following identities [34]:

$$\begin{aligned} S_B(p) &= \sqrt{2 + p^2 + 2\sqrt{1 + p^2u^2}} + \sqrt{2 + p^2 - 2\sqrt{1 + p^2u^2}} \\ &= \sqrt{4u^2 + (p + \sqrt{p^2 + 4(1 - u^2)})^2} \end{aligned} \quad (\text{A.1})$$

$$\begin{aligned} S_F(p) &= \frac{1}{\sqrt{2}}\sqrt{1 + 2p^2 + \sqrt{1 + 4p^2u^2}} + \frac{1}{\sqrt{2}}\sqrt{1 + 2p^2 - \sqrt{1 + 4p^2u^2}} \\ &= \sqrt{u^2 + (p + \sqrt{p^2 + (1 - u^2)})^2} . \end{aligned} \quad (\text{A.2})$$

Using a cutoff regularization for the integral over p and changing the integration variable $z = p + \sqrt{p^2 + 4(1 - u^2)}$ and $z = p + \sqrt{p^2 + (1 - u^2)}$, respectively, the integrals become

$$\begin{aligned} \int_0^L dp S_B(p) &= \int_{\sqrt{4(1-u^2)}}^{L+\sqrt{L^2+4-4u^2}} \frac{dz}{z} \left(\frac{4-4u^2}{z} + z \right) \sqrt{4u^2 + z^2} \\ \int_0^L dp S_F(p) &= \int_{\sqrt{1-u^2}}^{L+\sqrt{L^2+1-u^2}} \frac{dz}{z} \left(\frac{1-u^2}{z} + z \right) \sqrt{u^2 + z^2} \end{aligned} \quad (\text{A.3})$$

which can be straightforwardly evaluated.

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