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MUSICAL CONSONANCE AND CRITICAL BANDWIDTH

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1. Introduction

This year just a century ago H. von Helmholtz (1821-1894) published his book "Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik". His treatment of musical consonance, the central problem of the book, has proved to be the most acceptable explanation of the well-known fact that the consonant chords correspond with simple frequency ratios of the tones (octave 1:2, fifth 2:3, fourth 3:4, major third 4:5, major sixth 3:5> minor third 5:6, minor sixth 5:8).

Von Helmholtz's explanation was based on the hypothesis of Ohm (1787-1854) that the hearing organ performs a Fourier analysis of sound, so each periodical vibration is dispersed in its partials (fundamental tone plus overtones). After Von Helmholtz dissonance occurs when two or more partials differ so little in frequency that they cannot exist independently but interfere with each other (minimum consonance at 30-40 beats/sec). The more partials of a tone coincide with partials of another one, the less the chance on interferences is. This can explain the fact that simple frequency ratios give the most consonant chords.

Especially by Stumpf (1848-1936) many objections against these considerations were raised, for instance that after Von Helmholtz consonance alters by transposing a chord. Stumpf[1] introduced "fusion" ("Verschmelzung") as the origin of consonance: a chord is more consonant as it is easier perceived as one tone. Though undoubtedly a relation between consonance and fusion exists, at least for complex tones, this notion does not explain the consonance phenomenon at all as Révész[2] rightly stated.

During the last decennia the physiological knowledge of the hearing process, the physical equipment and the psychological methods of experiment are improved so much that a reconsideration of Von Helmholtz's explanation might be of value. This paper gives only a brief survey of research in this field carried out by the authors during the last years.

2. The connotation of musical consonance

As a preliminary to further research an exploratory investigation was made on the different modes of judgments of musical intervals. In this way it was tried to determine what subjects mean by their qualification of chords as consonant or dissonant. 23 combinations of two pure tones were used, all within the octave. The frequency ratios of the intervals could be described by the numbers 1....12, so the intervals were 1:2, 2:3, 3:4, 3:5, 4:5,........,7:12, 11:12. Only multiples of 85 cps were available as pure tones, so it was not possible to have all intervals of the same height. Ten intelligent subjects (non-musicians) judged these chords against ten 7-Points rating scales: high-low, sharp-round, beautiful-ugly, active-passive, consonant-dissonant, euphonious-diseuphonious, wide-narrow, sounds like one tone-sounds like more tones, tense-quiet, rough-smooth. The correlations between the scales were computed and the correlation matrix factor-analysed.

It appeared that all data could be summarized by assuming 3 basic dimensions, which we called height, evaluation and fusion. Evaluation was closely related to consonance, beautiful and euphonious; with other words: consonance had a definite meaning coinciding with the two other notions, not with fusion. This result was used in further experiments by circumscribing consonance as beautiful and euphonious.

3. Consonance as a function of the dlstanos between two pure tones

From the foregoing and other experiments it appeared that the degree of consonance is a function of the distance (frequenoy difference) between the two tones of a ohord, as far as pure tones are concerned. To investigate this relation more closely, a series of experiments were carried out in which teat subjects had to judge chords of two pure tones on consonance against a 7-points rating scale as in the experiments described above. To eliminate the factor height as good as possible, each subject judged only chords with a constant geometric mean. Different groups of 10-12 subjects were used to judge chords with a geometric mean of 125, 250, 500, 1000 and 2000 cps. The subjects were exposed to each interval 5 times and only the data of the subjects with a reasonably consistent judgment (a correlation above about 0.5 between the first and the last judgment of the intervals) sere maintained. The average scores at 125, 500 and 2000 cps are plotted in fig. 1.

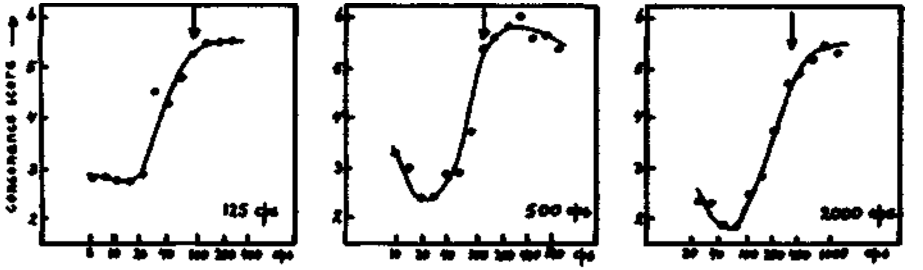

Fig. 1

The arrows indicate the critical bandwidth after Zwicker[3], derived from psychophysical hearing experiments. Fig.1 shows that the upper bend of the curves agrees very well with the critical band; the lower bend corresponds with about 0.2 x this bandwidth. In fig.2 this is demonstrated more explicitly.

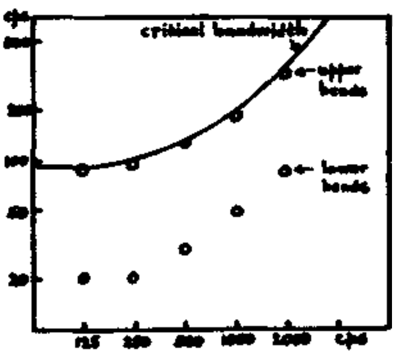

Fig. 2

This means that intervals with a frequency difference exceeding the critical bandwidth are judged as consonant; below this value the consonance appreciation falls rather sharp, reaching a mininum at about 0.2 x the critical bandwidth. For still smaller frequency differences we get slow beats which are appreciated core and more positively as the frequency difference decreases. This is shown in detail by a comparable experinent of Guthrie and Morrill4 at one frequency.

These results show that the critical band is an important factor in relation to consonance. We may state that this conclusion represents a valuable modification of Von Helmholtz's conception that dissonance is caused by interference of pure tones. Moreover it demonstrates that for pure tones consonance is not related to simple frequency ratios though for musicians the recognition of well-known intervals may affect their consonance appreciation of chords consisting of pure tones.

4. Consonance as a function of the distance between two complex tones

Using the results of the foregoing paragraph we can show that for complex tones (fundamental tone plus overtones) consonance is indeed related to simple frequency ratios. Plotting the consonance scores of fig.1 together with the corresponding data at 250 and 1000 cps as a function of the critical bandwidth: and averaging these curves we get a standard curve which is reproduced in fig.3. By a linear transformation the evaluation scale is replaced by a "consonance scale", 1 corresponding with maximum and 0.1 with minimum appreciation. For small frequency differences the curve is extended in accordance with the data of Guthrie and Morrill4.

We can use thi3 diagram to get some impression how consonance varies as a function of the distance (frequency difference of the fundamental tones) between two complex tones. As an illustration we take the case of a constant tone of 250 cps and a variable tone between 250 and 500 cps, each tone composed of the first 6 partials. We define as the consonance of the chord the sun of the dissonances (right scale of fig.3) of the intervals between each two adjacent partials, subtracted from 1. This consonance value is calculated as a function of the frequency of the variable tone and plotted in fig.4.

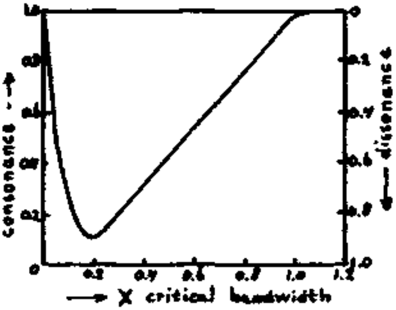

Fig. 3

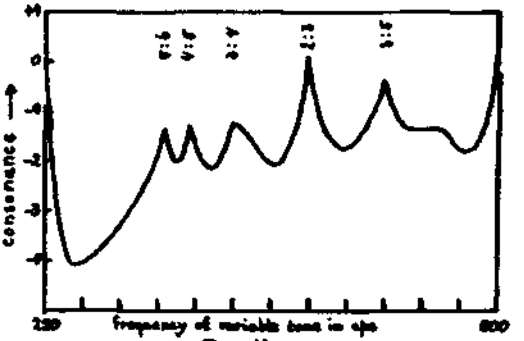

Fig. 4

interesting conclusions are justified:
a. The simple frequency ratios (1:1, 1:2, 2:3, 3:4, 3:5, 4:5, 5:6) are singular ooints of the curve, each being surrounded by a region of lower consonance. This explains that for complex tones consonance is related to simple frequency ratios.
b. The consonance of 1:1 and 1:2 is 1, which means that the distances between the adjacent partials exceed the critical bandwidth. It appears that this only applies for 6 and fewer partials; this is in agreement with the fact, also underlined by Von Helmholtz, that tones with strong higher partials sound sharp and dissonant.
c. A small deviation from a simple frequency ratio has much more influence on the unison (1:1), octave and fifth than on the other consonant intervals. This corresponds with the practice of tuning musical instruments by octave and fifth relations. In connection with this the fact is important that the impure thirds of the equi-tempered intonation are much better tolerated than impure octaves and fifths would have been (compare the position of the maximum of the curve with the vertical dashes in fig.4, giving the intervals of the equi-tempered intonation).
d. The octave and fifth are the most consonant intervals, which is in agreement with practice.

Fig.5 illustrates how the consonance of simple frequency ratios varies with frequency. The curves show that the mutual relations of consonance of the different intervals are nearly independent of frequency. The fall at low frequencies is caused by the fact that below 500 cps the critical bandwidth is nearly constant and the fall at high frequencies by the fact that in this range the critical bandwidth increases more rapid than frequency. Tne curves of fig.5 are important in relation to the controversy

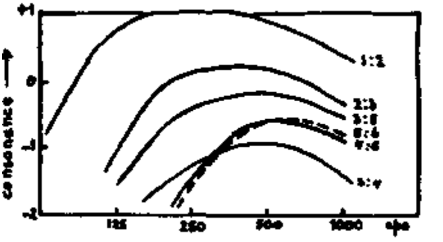

Fig. 5

Von Helmholtz-Stumpf about the dependence of consonance on frequency. After Von Helmholtz the consonance of an interval increases with frequency, after Stumpf con- ----nce is independent of frequency. Von Helmholtz seems to be right below about 250 cpa and Stumpf in a range of 1-? octaves above this frequency.

5. The distance between partials in music

It is of interest to examine to which extent the relation of consonance to critical bandwidth can be found back in musical compositions. Only some prelimi- nary investigations are made and one of the most interesting results will be shown.

We analysed the chords of compositions in the following way. After choosing a note, for instance b^1(494 cps), we determined which fraction of its total time of presentation this note sounds together with a tone at a distance of 1/2 tone (b^1-flat or a^1 sharp and c^2), which fraction together with a tone at a distance of 2/2 tone, etc. The result of such an analysis we called the interval distribution of b^1. Such interval distributions can be calculated for different tones and also taking into account the presence of overtones.

In figure 6 the mean value of interval distributions of tones around 125, 250, 500, 1000 and 2000 cps is plotted as a function of the number of partials taken into account. The interval distributions were calculated from all chords of 4 chorals of J.S.Bach (St. Matthew Passion, nos. 3, 16, 23 and 31). The ordinate is given in cps, so the graph shows the "density" of partials (mean frequency difference between adjacent partials) at a number of frequencies. As we may expect, introducing more partials affects more the "density" at high frequencies than at low frequencies. In fig.7 the asymptotic values of the curves of fig.6 are plotted as a function of frequency together with the critical bandwidth. It is very interesting that the curves are almost parallel, demonstrating that the composer selected his chords in such a way that on the average over the whole frequency range the critical band is "penetrated" to the same degree.

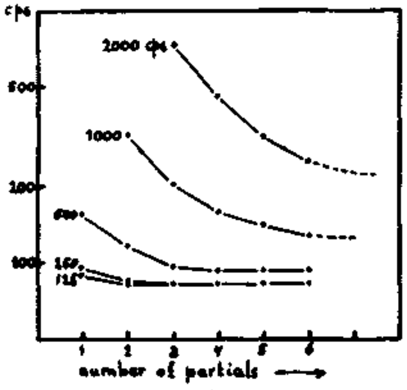

Fig. 6

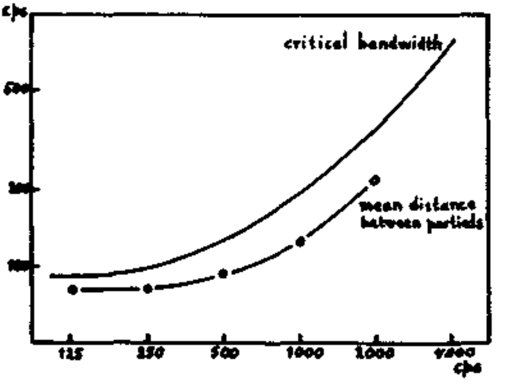

Fig. 7

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