Observational Constraints on Loop Quantum Cosmology

Martin Bojowald,[1] Gianluca Calcagni,[2] and Shinji Tsujikawa[3]

[1]*Institute for Gravitation and the Cosmos, The Pennsylvania State University, 104 Davey Lab, University Park, Pennsylvania 16802, USA*

[2]*Max Planck Institute for Gravitational Physics, (Albert Einstein Institute), Am Mühlenberg 1, D-14476 Golm, Germany*

[3]*Department of Physics, Faculty of Science, Tokyo University of Science, 1-3, Kagurazaka, Shinjuku-ku, Tokyo 162-8601, Japan*

(Received 27 January 2011; revised manuscript received 27 August 2011; published 15 November 2011)

In the inflationary scenario of loop quantum cosmology in the presence of inverse-volume corrections, we give analytic formulas for the power spectra of scalar and tensor perturbations convenient to compare with observations. Since inverse-volume corrections can provide strong contributions to the running spectral indices, inclusion of terms higher than the second-order runnings in the power spectra is crucially important. Using the recent data of cosmic microwave background and other cosmological experiments, we place bounds on the quantum corrections.

DOI: 10.1103/PhysRevLett.107.211302 PACS numbers: 98.80.Es, 04.60.Pp, 98.80.Cq

One of the motivations to search for a quantum theory of gravity is the desire to unify general relativity with quantum mechanics and, thereby, resolve classical singularities such as the big bang or those associated with black holes. Observational implications of quantum gravity, however, present a delicate issue. Based on dimensional grounds, cosmology in a nearly isotropic setting seems to allow quantum corrections only as powers of the small quantity $\ell_{\rm Pl}H \approx 10^{-10}$, where $\ell_{\rm Pl}$ is the Planck length and $H^{-1} = a/\dot{a}$ is the Hubble radius (a is the scale factor in the flat Friedmann-Robertson-Walker background and dots denote derivatives with respect to cosmic time t). This dimensional argument is supported by low-energy effective actions of higher-curvature type.

Dimensional arguments, generally, are overcome if there are more than two dynamical scales of the same dimension. Detailed physics rather than rough estimates are then required to determine which geometric mean of the scales is relevant in a given regime. In cosmology, an additional distance scale L would allow a multitude of dimensionless combinations $\ell_{\rm Pl}^{\alpha}H^{\beta}L^{\gamma}$ with $\alpha - \beta + \gamma = 0$, not all of them small. Quantum gravity provides ample motivation for the existence of a third scale by suggesting discrete spatial structures. While the discreteness scale L is often expected to be near $\ell_{\rm Pl}$, it is not identical to it and also depends on excitation levels of states (rather than just Newton's and Planck's constants).

One explicit formulation of such a discrete version of gravity is loop quantum gravity (LQG) [1]. Discreteness arises on the space of metrics (geometrical operators acquiring discrete spectra). In a nearly homogeneous quantum space-time, one can think of any region of volume V to consist of discrete patches, each roughly of size L^3 with the length L determined by an underlying quantum-gravity state. Discrete spectra imply that derivatives by L, as they ubiquitously appear in canonical expressions via Poisson brackets, are replaced by finite difference quotients. As a simple example for so-called inverse-volume corrections, $(2\sqrt{L})^{-1} = d\sqrt{L}/dL$ would, when evaluated for discrete operators, become $(\sqrt{L + \ell_{\rm Pl}} - \sqrt{L - \ell_{\rm Pl}})/2\ell_{\rm Pl}$, which strongly differs from $(2\sqrt{L})^{-1}$ for $L \sim \ell_{\rm Pl}$. For larger L, corrections are perturbative and of the order $\ell_{\rm Pl}/L$; no factor of H appears. The ratio $\ell_{\rm Pl}/L$ can easily be much larger than $\ell_{\rm Pl}H$, explaining why this type of discreteness could give rise to stronger quantum effects.

The results of detailed constructions in LQG, following [2,3], will be summarized momentarily. First, we emphasize that the discreteness does not break general covariance in the equations used here (assuming small corrections). This has been demonstrated by an elaborate analysis of the gauge contents of the quantum-corrected theory, verifying the existence of a closed algebra of gauge generators [4]. Covariance, and the space-time structure it belongs to, is then not destroyed but deformed. (Deformations of classical symmetries play an important role in several approaches to quantum-gravity phenomenology [5]. The deformations considered here are on a different footing, however, because they do not refer just to Poincaré transformations of Minkowski space.)

Here, using currently available data, we place constraints on inverse-volume corrections for inflation. Since scalar and tensor perturbations are subject to strong modifications of the power on large scales, the corrections are bounded from above. A detection of gravitational waves and the precise measurement of cosmic microwave background (CMB) anisotropies in future observations such as Planck will potentially allow us to make a decisive test for loop quantum cosmology (LQC) inflation.

A simplified implementation of corrections expected from LQG in cosmological scenarios via perturbations around homogeneous or other reduced models can be achieved in LQC [6]. With a phenomenological approach to effective dynamics, the cosmological equations can be summarized in a single Mukhanov equation for the gauge-invariant scalar perturbation u_k, $u_k'' + (s^2k^2 - z''/z)u_k = 0$ [3] in momentum space with the comoving wave number k, where primes denote derivatives with respect to conformal time $\tau = \int a^{-1}dt$. Similarly, tensor modes are subject to the equation $w_k'' + (\alpha^2k^2 - \tilde{a}''/\tilde{a})w_k = 0$ [7]. Here, $z(a, \varphi)$ and $\tilde{a}(a)$ are background functions and $\alpha^2 \approx 1 + 2\alpha_0\delta_{\rm Pl}$ and $s^2 = 1 + \chi\delta_{\rm Pl}$ are the propagation speeds squared, differing from the speed of light by quantum corrections.

The quantum corrections are characterized by (i) numerical coefficients α_0 and χ and (ii) the function $\delta_{\rm Pl} \propto a^{-\sigma}$ determining the size of inverse-volume corrections. The values of α_0, χ, and σ are currently subject to quantization ambiguities. χ is parametrized as $\chi = \sigma\nu_0(\sigma/6 + 1)/3 + \alpha_0(5 - \sigma/3)/2$, where ν_0 is related to α_0 and σ by the consistency condition [3]

$$\nu_0(\sigma - 3) = 3\alpha_0(\sigma - 6)/(\sigma + 6). \tag{1}$$

While σ takes values in the range $0 < \sigma \leq 6$, the size of $\delta_{\rm Pl}$ does not depend on the values of α_0 and ν_0. With $\sigma > 0$, $\delta_{\rm Pl}$ is larger at early times, in agreement with discreteness departing from the Planck scale in a more classical universe. The aim of this Letter is to restrict $\delta_{\rm Pl}$ by observations. We will mainly place bounds on the combination $\alpha_0\delta_{\rm Pl}$ during slow-roll (SR) inflation, for which the precise origin of α_0 and ν_0 or the scale hidden in $\delta_{\rm Pl}$ is not essential.

Corrections in the evolution equations arise only in the k^2 term, not in the time derivative of the d'Alembertian, yet they are covariant according to the corrected gauge transformations [4]. Thus, one typical assumption of higher-curvature theories is violated. Moreover, the propagation speed of tensor modes differs from the scalar one since in general $2\alpha_0 \neq \chi$. Again, this is only possible with the change in the underlying manifold and gauge structure, and gives rise to additional characteristic effects. With different types of equations for scalar and tensor modes, there are changes to the standard inflationary spectra and the tensor-to-scalar ratio.

In Ref. [3], two of us evaluated the inflationary observables in terms of the three SR parameters $\epsilon = -\dot{H}/H^2$, $\eta = -\ddot{\varphi}/(H\dot{\varphi})$, and $\xi^2 = (\dddot{\varphi}/\dot{\varphi})/H^2$, where φ is a scalar field with potential $V(\varphi)$. In order to place observational bounds on concrete inflaton potentials, it is more convenient to use SR parameters expressed by V and its derivatives: $\epsilon_V \equiv \kappa^{-2}(V_{,\varphi}/V)^2/2$, $\eta_V \equiv \kappa^{-2}V_{,\varphi\varphi}/V$, $\xi_V^2 \equiv \kappa^{-4}V_{,\varphi}V_{,\varphi\varphi\varphi}/V^2$, where $\kappa^2 = 8\pi G$ (G is the gravitational constant). For conversion formulas from ϵ, η, ξ^2 to ϵ_V, η_V, ξ_V^2 and all the technical details, we refer to [8], together with a discussion of cosmic variance.

The power spectra of scalar and tensor perturbations, evaluated at the Hubble horizon crossing during inflation ($k \approx aH$), are given, respectively, by [3]

$$\mathcal{P}_s = \frac{GH^2}{\pi\epsilon}(1 + \gamma_s\delta_{\rm Pl}), \qquad \mathcal{P}_t = \frac{16GH^2}{\pi}(1 + \gamma_t\delta_{\rm Pl}), \tag{2}$$

where $\gamma_s = \nu_0(\sigma/6 + 1) + \sigma\alpha_0/(2\epsilon) - [\sigma\nu_0(\sigma + 6) + 3\alpha_0(15 - \sigma)]/[18(\sigma + 1)]$ and $\gamma_t = (\sigma - 1)\alpha_0/(\sigma + 1)$. We expand the scalar spectrum about a pivot wave number k_0, as

$$\ln\mathcal{P}_s(k) = \ln\mathcal{P}_s(k_0) + [n_s(k_0) - 1]x + \frac{\alpha_s(k_0)}{2}x^2 + \sum_{m=3}^{\infty}\frac{\alpha_s^{(m)}(k_0)}{m!}x^m, \tag{3}$$

where $x = \ln(k/k_0)$, $n_s(k) - 1 \equiv d\ln\mathcal{P}_s(k)/d\ln k$, and $\alpha_s^{(m)}(k) \equiv d^{m-2}\alpha_s/(d\ln k)^{m-2}$. The tensor spectrum can be expanded in a similar way with a different index $n_t(k) \equiv d\ln\mathcal{P}_t(k)/d\ln k$. While such expansions to second order are standard in cosmology, terms of order higher than 2 will become important in our analysis.

The spectral indices are

$$n_s - 1 = -6\epsilon_V + 2\eta_V - c_{n_s}\delta_{\rm Pl}, \quad n_t = -2\epsilon_V - c_{n_t}\delta_{\rm Pl}, \tag{4}$$

with quantum-gravity corrections $c_{n_{s,t}} = f_{s,t} + \cdots$ whose dominant contributions are $f_s \equiv \sigma[3\alpha_0(13\sigma - 3) + \nu_0\sigma(6 + 11\sigma)]/[18(\sigma + 1)]$ and $f_t \equiv 2\sigma^2\alpha_0/(\sigma + 1)$. For $\sigma \gtrsim O(1)$ the variation of $\delta_{\rm Pl}$ is fast ($\delta_{\rm Pl} \propto a^{-\sigma} \propto k^{-\sigma}$ at Hubble crossing), so that $f_{s,t}$ provide dominant contributions to the scalar and tensor runnings as well, $\alpha_{s,t}(k_0) \equiv dn_{s,t}/d\ln k|_{k=k_0} \approx \sigma f_{s,t}\delta_{\rm Pl}(k_0)$. Similarly, the mth order terms are $\alpha_{s,t}^{(m)}(k_0) \approx (-1)^m\sigma^{m-1}f_{s,t}\delta_{\rm Pl}(k_0)$, and hence we can evaluate the sum in Eq. (3) as

$$\sum_{m=3}^{\infty}\frac{\alpha_{s,t}^{(m)}}{m!}x^m = \left[x\left(1 - \frac{1}{2}\sigma x\right) + \frac{e^{-\sigma x} - 1}{\sigma}\right]f_{s,t}\delta_{\rm Pl}. \tag{5}$$

This expression is valid for any value of σ and of the pivot scale k_0 within the observational range of CMB. Since the LQC corrections to the runnings $\alpha_{s,t}$ can be large, inclusion of the higher-order terms (5) is important to estimate the power spectra properly.

For the CMB likelihood analysis we also take into account the second-order terms of slow-roll parameters, i.e., $\alpha_s = -24\epsilon_V^2 + 16\epsilon_V\eta_V - 2\xi_V^2 + c_{\alpha_s}\delta_{\rm Pl}$ and $\alpha_t = -4\epsilon_V(2\epsilon_V - \eta_V) + c_{\alpha_t}\delta_{\rm Pl}$, where the dominant contributions to $c_{\alpha_{s,t}}$ correspond to $c_{\alpha_{s,t}} \approx \sigma f_{s,t}$. In the numerical code, the full expressions of the coefficients $c_{n_{s,t}}$ and $c_{\alpha_{s,t}}$ [8] are used. At the pivot scale k_0 we have the tensor-to-scalar ratio $r(k_0) \equiv \mathcal{P}_t(k_0)/\mathcal{P}_s(k_0) = 16\epsilon_V(k_0) + c_r\delta_{\rm Pl}(k_0)$, where $c_r = 8[3\alpha_0(3 + 5\sigma + 6\sigma^2) - \nu_0\sigma(6 + 11\sigma)]\epsilon_V(k_0)/[9(\sigma + 1)] - 16\sigma\alpha_0\eta_V(k_0)/3$.

In the quasi-de Sitter background, $\delta_{\rm Pl} \propto k^{-\sigma}$ gives the relation $\delta_{\rm Pl}(k) \approx \delta_{\rm Pl}(k_0)(k/k_0)^{-\sigma} = \delta_{\rm Pl}(\ell_0)(\ell/\ell_0)^{-\sigma}$, where ℓ are the CMB multipoles related to k via

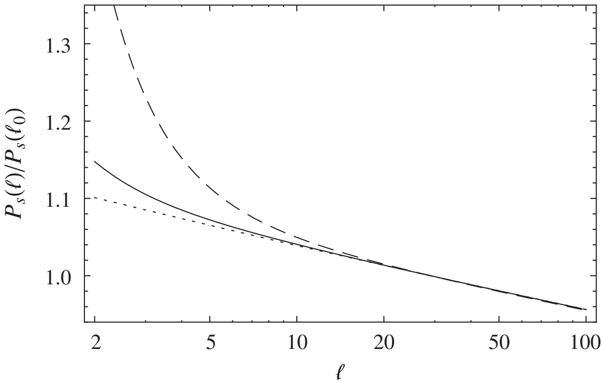

FIG. 1. Primordial scalar power spectrum $\mathcal{P}_s(\ell)$ for the case $n = 2$, $\sigma = 2$, and $\epsilon_V(k_0) = 0.009$, with three different values of $\delta(k_0)$: 0 (classical case, dotted line), 7×10^{-5} (experimental upper bound, solid line), 4.8×10^{-4} (1/10 of the *a priori* upper bound, dashed line). Here the pivot wave number is $k_0 = 0.002\ \mathrm{Mpc}^{-1}$, which corresponds to $\ell_0 = 29$.

$k \approx (h/10^4)\ell\ \mathrm{Mpc}^{-1}$ ($h \approx 0.7$ is the reduced Hubble constant). With the large-volume expansion of quantum corrections, we require that $\delta_{\rm Pl}(k) \ll 1$ at all scales. For $\sigma > 0$ the LQC correction is most significant on the largest scales observed in the CMB ($\ell = 2$). This property can be clearly seen in Fig. 1, where the pivot scale for the scalar power spectrum is taken to be $\ell_0 = 29$. Intuitively, this happens because the largest cosmological scales correspond to those where and when space-time quantum effects were larger, while smaller scales have been affected by ordinary physics. Imposing the condition $\delta_{\rm Pl}(\ell = 2) \ll 1$, this gives

$$\delta_{\rm Pl}(\ell_0) \ll (2/\ell_0)^\sigma \quad (6)$$

at the multipole ℓ_0. For larger σ and ℓ_0, $\delta_{\rm Pl}(\ell_0)$ is constrained to be smaller. We assume that inflation completely takes place in the large-volume regime, and that longer-wavelength modes that might violate the bound (6) do not enter the inflationary Fourier analysis.

For concreteness, let us consider the power-law potential $V(\varphi) = \lambda\varphi^n$, for which $\epsilon_V = n^2/(2\kappa^2\varphi^2)$ and

$$\eta_V = \frac{2(n-1)}{n}\epsilon_V, \qquad \xi_V^2 = \frac{4(n-1)(n-2)}{n^2}\epsilon_V^2. \quad (7)$$

Among the variables σ, α_0, and ν_0 we have the relation (1), a condition under which, for given n and σ, the inflationary observables can be expressed via ϵ_V and $\delta \equiv \alpha_0\delta_{\rm Pl}$ for $\sigma \neq 3$, or by ϵ_V and $\tilde{\delta} \equiv \nu_0\delta_{\rm Pl}$ for $\sigma = 3$.

We carry out the CMB likelihood analysis by varying the parameters ϵ_V and δ in the cosmological Monte Carlo (COSMOMC) code [9]. We use the 7-year WMAP data combined with large-scale structure, the Hubble constant measurement from the Hubble Space Telescope, supernovae type Ia, and big bang nucleosynthesis [10]. We assume the flat Λ-cold dark matter model with no fraction of massive neutrinos in the dark matter density ($f_\nu = 0$).

In the likelihood analysis, we vary the following eight parameters: (i) baryon density today, Ω_b, (ii) dark matter density today, Ω_c, (iii) the ratio of the sound horizon to the angular diameter distance, θ, (iv) the reionization optical depth, τ, (v) $\delta(k_0)$, (vi) $\epsilon_V(k_0)$, (vii) $\mathcal{P}_s(k_0)$, and (viii) the Sunyaev-Zel'dovich amplitude, $A_{\rm SZ}$. We take the pivot wave number $k_0 = 0.002\ \mathrm{Mpc}^{-1}$ ($\ell_0 \approx 29$) used by the WMAP team. $\delta(k_0)$ and $\epsilon_V(k_0)$ are constrained at this scale. While the bound on δ depends on the pivot scale (and it tends to be smaller for larger k_0), that on $(k_0)^\sigma\delta(k_0)$ does not.

While we assume a standard treatment of the reionization with a smooth interpolation, more general reionization scenarios can potentially affect constraints on observables especially for $n_s > 1$ [11]. The analysis in [11] shows that the allowed region with $n_s < 1$ is not strongly modified, which is the case for our potentials.

The exponential term $e^{-\sigma x} = (k_0/k)^\sigma$ in Eq. (5) gives rise to the enhancement of the power spectra on large scales, as we see in Fig. 1. In this sense the LQC corrections can be distinguished from the suppression effects coming from the noncommutative geometry or string corrections [12]. For $\sigma \gtrsim 3$, the growth of the term $e^{-\sigma x}$ is so significant that $\delta_{\rm Pl}(\ell)$ must be very much smaller than 1 for most of the scales observed in the CMB, in order to satisfy the bound $\delta_{\rm Pl}(\ell = 2) \ll 1$. More precisely, LQC corrections manifest themselves mainly at $\ell = 2, 3$, where cosmic variance dominates, so it seems implausible to isolate these effects. For $\sigma < 3$, the LQC modification to the classical power spectra also affects larger multipoles ℓ, and hence it is possible to constrain it from CMB anisotropies.

In Fig. 2 we plot the 2D posterior distributions on the parameters $\delta(k_0)$ and $\epsilon_V(k_0)$ with $k_0 = 0.002\ \mathrm{Mpc}^{-1}$ for $n = 2$ and $\sigma = 2$. The two parameters are constrained to be $\delta(k_0) < 6.7 \times 10^{-5}$ and $\epsilon_V(k_0) < 0.013$ (95% C.L.). The modification of the large-scale power spectra ($\ell \lesssim 20$) shown in Fig. 1 leads to the upper bound on $\delta(k_0)$. The condition (6) gives the prior $\delta_{\rm Pl}(\ell_0) \ll 4.8 \times 10^{-3}$ at $\ell_0 = 29$, so that for $\alpha_0 = O(1)$ the observational bound is smaller by 2 orders of magnitude. For larger k_0 the observational upper bounds on $\delta(k_0)$ tend to be smaller for given σ. For $k_0 = 0.05\ \mathrm{Mpc}^{-1}$ and $\sigma = 2$, we find that $\delta(k_0) < 1.2 \times 10^{-7}$ (95% C.L.), in which case the theoretical expected amplitude [$\delta_{\rm Pl}(k_0) \sim 10^{-8}$ or a few orders of magnitude higher [3]] can be accessible.

For smaller σ the observational upper bound on $\delta(k_0)$ tends to be larger, with milder enhancement of the power spectra on large scales. In Fig. 3 we show the likelihood results for $\sigma = 1$, in which case the LQC correction is constrained to be $\delta(k_0) < 3.6 \times 10^{-2}$ (95% C.L.). Meanwhile, the *a priori* criterion (6) gives $\delta_{\rm Pl}(k_0) \ll 6.9 \times 10^{-2}$. For $\alpha_0 = O(1)$, the case $\sigma = 1$ is marginally consistent with the combined SR/$\delta_{\rm Pl}$ truncation.

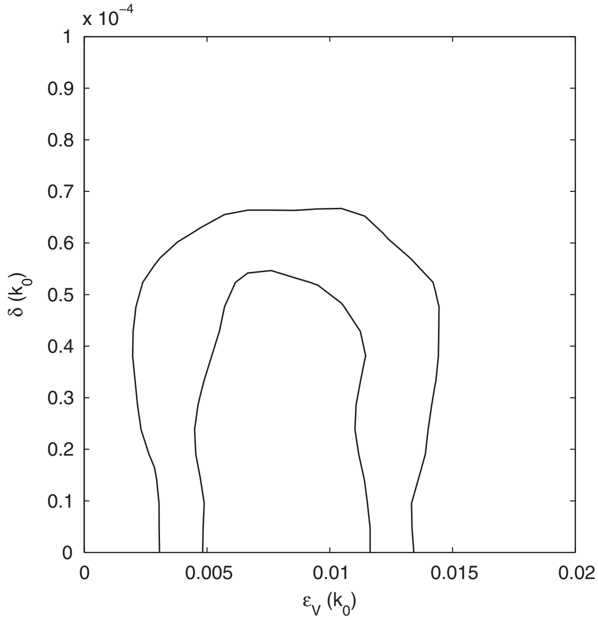

FIG. 2. 2D marginalized distribution for the quantum-gravity parameter $\delta(k_0) = \alpha_0 \delta_{\rm Pl}(k_0)$ and the slow-roll parameter $\epsilon_V(k_0)$ with the pivot $k_0 = 0.002~{\rm Mpc}^{-1}$ for $n = 2$ and $\sigma = 2$. The internal and external solid lines correspond to the 68% and 95% confidence levels, respectively.

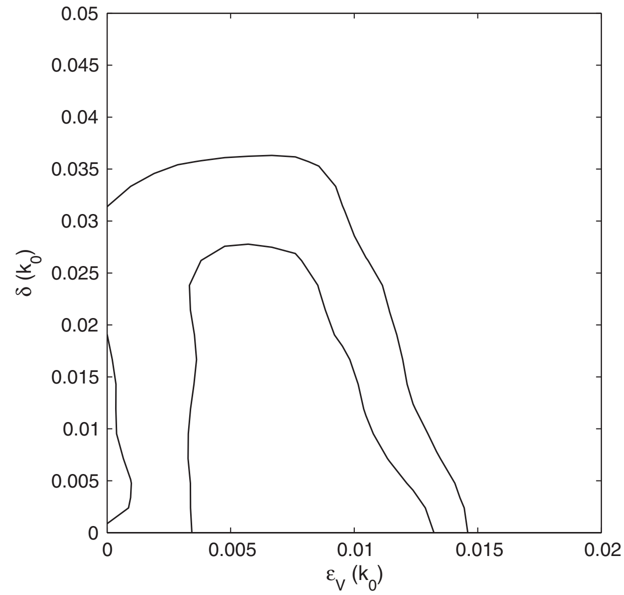

FIG. 3. 2D marginalized distributions as in Fig. 2, but for the case $n = 2$ and $\sigma = 1$.

For $\sigma \lesssim 1$, the exponential factor $e^{-\sigma x}$ does not change rapidly with smaller values of $f_{s,t}$, so that the LQC effect on the power spectra would not be very significant even if $\delta(k_0)$ was as large as $\epsilon_V(k_0)$. Our likelihood analysis shows that the observational upper bound on $\delta(k_0)$ exceeds the *a priori* upper limit of $\delta_{\rm Pl}(k_0)$ given by Eq. (6). Since $\delta(k_0)$ can be as large as 1, the validity of the approximation $\delta(k_0) < \epsilon_V(k_0)$ used in the main formulas may break down in such cases.

Under the conditions $\epsilon_V, \delta \ll 1$, it follows that $\epsilon_V \approx (\kappa^2/2)(\dot{\varphi}/H)^2$. Then the number of e-foldings during inflation is given by $N \equiv \int_t^{t_f} d\tilde{t} H \approx \kappa \int_{\varphi_f}^{\varphi} d\tilde{\varphi}/\sqrt{2\epsilon_V(\tilde{\varphi})}$, where φ_f is the field value at the end of inflation [determined by the condition $\epsilon_V \approx O(1)$]. For the power-law potentials one has $N \approx n/(4\epsilon_V) - n/4$, which gives $\epsilon_V \approx n/(4N + n)$. For $n = 2$, the theoretically constrained range $45 < N < 65$ corresponds to $0.008 < \epsilon_V < 0.011$. The probability distributions of ϵ_V in Figs. 2 and 3 are consistent with this range even in the presence of the LQC corrections, so the quadratic potential is compatible with observations as in standard cosmology.

In summary, in inflation combined with LQC inverse-volume corrections we provided general formulas for the scalar and tensor power spectra and placed observational bounds on the size of corrections for a quadratic potential. In [8] we ran the COSMOMC code also for other potentials such as $V \propto \varphi^4$ and $V \propto e^{-\lambda\varphi}$ (for which the inflationary observables reduce, again, to δ and ϵ_V). We found that the observational upper bounds are practically independent of the inflaton potentials. This is because the LQC correction is approximately given by $\delta_{\rm Pl}(k) \approx \delta_{\rm Pl}(k_0)(k/k_0)^{-\sigma}$, which only depends on σ and the pivot scale k_0. Interesting and nontrivial effects do arise from the modified space-time structure underlying the dynamics. Even though quantum-geometry corrections are small, they can significantly change the runnings of spectral indices. Thus, the observational bounds on $\delta_{\rm Pl}$ can be much closer to theoretical values [$O(10^{-8})$] than often thought in quantum gravity. Our new techniques set the stage for systematic and stringent phenomenological evaluations.

M. B. was supported by NSF Grant No. PHY0748336. S. T. thanks JSPS for financial support (Grants No. 30318802 and No. 21111006).

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