Detecting groups of coherent voxels in fMRI data using spectral analysis and replicator dynamics

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Introduction

- **Motivation:** Investigation of coherent regions in the human brain analyzing the relationship between fMRI time series.
- **Idea:** Combining the approaches of spectral analysis and replicator dynamics for detecting groups of coherent voxels in fMRI data

Figure 1: Two axial slices of an individual subject showing members of the first (red) and second (yellow) coherent group as determined by the replicator process. On the right, the coherence matrix **C** is shown. The entries corresponding to voxels in the first and the second coherent group are again marked in red and yellow, resp.

Methods

- The relationship between fMRI time series can be described in terms of the coherence, a bivariate measure resulting from spectral analysis (Müller et al., 2001; Sun et al., 2004).
- When considering the whole brain, the coherence values are placed in a spectral matrix **C** which encodes the relationship between all fMRI time series.
- So far, groups of coherent voxels were determined from **C** by choosing a reference voxel *j* and determining all voxels *k* whose coherence with *j* exceeds a given threshold ρ_0, i.e. $[\mathbf{C}]_{jk} = \rho_{jk} > \rho_0$ (Müller et al., 2001).
- **New approach:** Using the spectral coherence matrix **C**, a replicator process is able to determine groups of voxels with the property that each voxel in the group exhibits a high coherence with every other group member (Lohmann and Bohn, 2002).

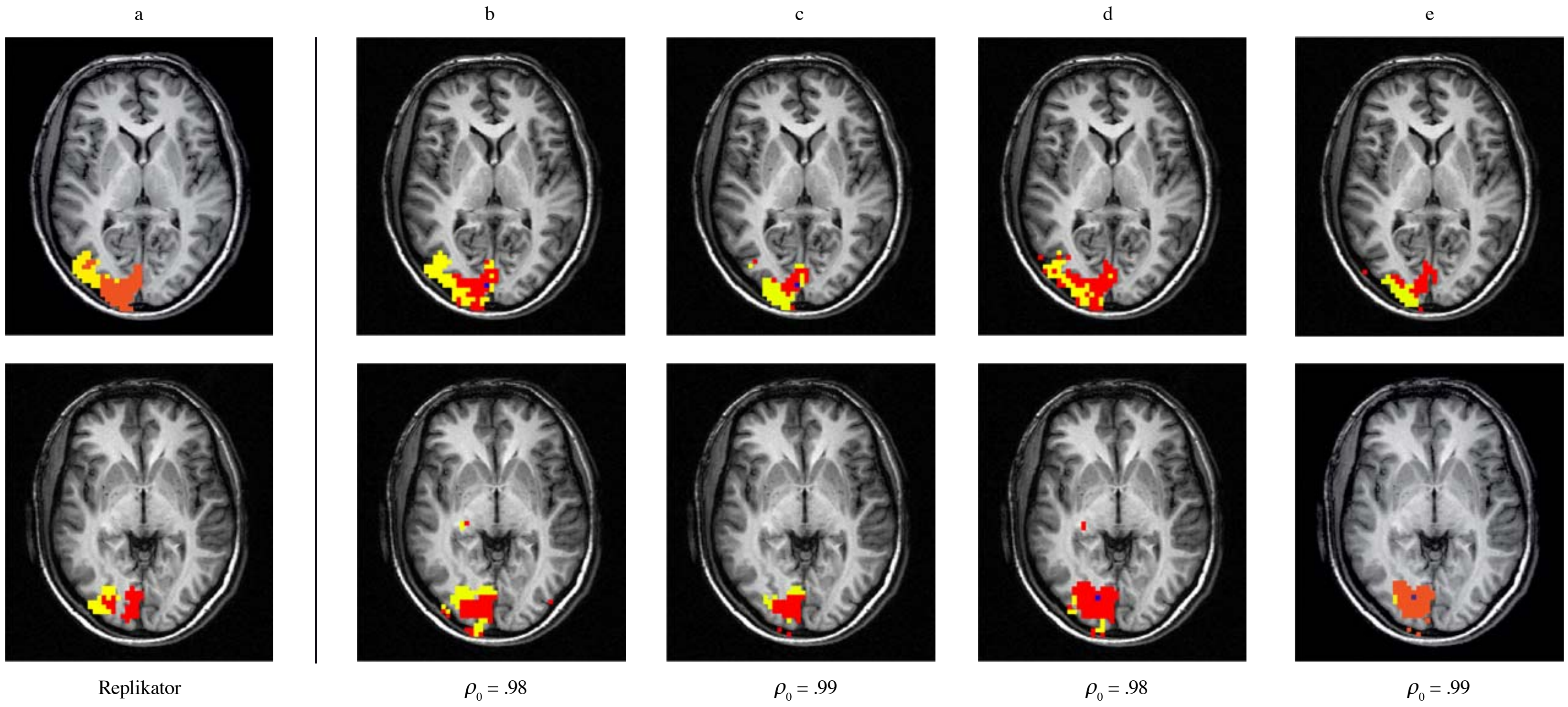

Figure 2: Axial slices showing groups of coherent voxels determined by spectral analysis and replicator dynamics (column a), and by selecting a reference voxel and an additional threshold ρ_0 (columns b-e). In the latter case, results critically depend on the choice of threshold (compare column b with c and d with e) and on the choice of the reference voxel (compare b with d and c with e). The reference voxel is marked in blue.

Coherence

- Let $X_k(t)$ be the fMRI time course of voxel *k*. Then

$$\mathbf{X}(t) = \begin{bmatrix} X_1(t) \\ \vdots \\ X_n(t) \end{bmatrix}$$

  is a multivariate weakly stationary stochastic process if

$$E[X_j(t)] = m_j$$

  and

$$E[X_j(\tau + t)X_k(t)] = E[X_j(\tau)X_k(0)] .$$

- For weakly stationary processes, the cross-covariance can be estimated via

$$C_{jk}(s) = \begin{cases} \frac{1}{l}\sum_{t=1}^{l-s} X_j(s+t)X_k(t) & , \quad s = 0,1,...,l-1 \\ C_{kj}(-s) & , \quad s = -1,...,-l+1 \end{cases} .$$

- Using a window $m < l$ and a phase-lag generator ω, the cross-spectral density can be estimated via

$$f_{jk}(\lambda) = \frac{1}{2\pi}\sum_{s=-m}^{m} exp(-i\lambda s)\omega(\frac{1}{m})C_{jk}(s) .$$

- The cross-coherence is the normalized modulus of the spectral density

$$0 \leq \rho_{jk}(\lambda) = |f_{jk}(\lambda)| / |f_{jj}(\lambda) f_{kk}(\lambda)|^{\frac{1}{2}} \leq 1.$$

- The cross-coherence $\rho_{jk}(\lambda)$ is a measure of the degree of linear association between time series and can be interpreted quantitatively. It can be represented as a matrix

$$\mathbf{C} = [\rho_{jk}]_{j,k=1,...,n} .$$

- The number of the effective degrees of freedom *d* depends on the phase-lag generator ω, and on the size of the window $m < l$

$$d^{-1} = \frac{1}{2l}\sum_{s=-m}^{m} \omega(\frac{s}{m}) .$$

- For a large number of effective degrees of freedom ($d > 20$), the random variable

$$\phi_{jk}(\lambda) = tanh^{-1}(\rho_{jk}(\lambda))$$

  is approximately normally distributed (Enochson and Goodman, 1965).

- The lower and upper bound of the coherence confidence interval is

$$tanh\left\{\phi_{jk}(\lambda) \pm \frac{u_{\alpha/2}(d-2)^{\frac{1}{2}} - 1}{d-2}\right\} ,$$

  where $u_{\alpha/2}$ is taken from the standard normal distribution $\mathcal{N}_{0,1}$.

REFERENCES:

Enochson LD, Goodman NR. 1965. Gaussian approximation to the distribution of sample coherence. *Res. and Technol. Div., AFSC, Wright-Patterson AFB, OH. AFFDL* TR 65-67.
Fisher RA. 1930. The genetical theory of natural selection. *Oxford University Press,* London.
Lohmann G, Bohn S. 2002. Using replicator dynamics for analyzing fMRI data of the human brain. *IEEE Trans Med Imag* 21(5), 485-492.
Müller K, Lohmann G, Bosch V, von Cramon DY. 2001. On multivariate spectral analysis of fMRI time series.*NeuroImage* 14(2), 347-356.
Sun, FT, Miller, LM, D'Esposito, M. 2004. Measuring interregional functional connectivity using coherence and partial coherence analyses of fMRI data. *NeuroImage* 21(2), 647-658.

Replicator Dynamics

- Discrete replicator equation

$$x_i(t+1) = x_i(t)\frac{(\mathbf{C}\mathbf{x}(t))_i}{\mathbf{x}(t)^T\mathbf{C}\mathbf{x}(t)}$$

$i = 1...n$ replicators, entities in an evolutionary process

C matrix of the *fitness* of all replicators
measure of successful interaction with other replicators

x_i *proportion* of replicator *i*
indicating the percentage of the population that is identical to *i*

$x^T\mathbf{C}x$ mean fitness of the population

- The dynamics of such replicator systems are described by the fundamental theorem of natural selection (Fisher 1930).
- Starting from an initial vector x the replicator process evolves toward a vector x which maximizes $x^T\mathbf{C}x$.
- Replicators with good fitness are assigned a high proportion.
- When approaching a stationary point, a dominant network of replicators with the best fitness and proportion values emerges.

Application

- Spectral analysis was applied to fMRI data resulting from a right hemifield stimulation.
- Coherence values ρ_{jk} were estimated using the normalized modulus of the spectral density computed for the frequency of the experimental task.
- The resulting coherence matrix **C** was subjected to a replicator process in order to detect the group of maximally coherent voxels.
- A second coherent group was determined from **C** after voxels belonging to the first group were deleted.

Conclusion

- Using coherence matrices, replicator processes can be used to find coherent voxels.
- To detect groups of coherent voxels, the approach of replicator dynamics is parameter-free and does not require the a priori selection of a reference voxel.
- In contrast to the selection of a reference voxel, the new approach is able to detect groups of coherent voxels in the way that each voxel in the group shows a high coherence with every other group member.