Bonus Yangian Symmetry for the Planar S Matrix of $\mathcal{N}=4$ Supersymmetric Yang-Mills Theory

Niklas Beisert* and Burkhard U. W. Schwab†

Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut), Am Mühlenberg 1, 14476 Potsdam, Germany

(Received 17 March 2011; published 7 June 2011)

Recent developments in the determination of the planar S matrix of $\mathcal{N}=4$ super Yang-Mills are closely related to its Yangian symmetry. Here we provide evidence for a yet unobserved additional symmetry: the Yangian level-one helicity operator.

DOI: 10.1103/PhysRevLett.106.231602 PACS numbers: 11.25.Tq, 11.30.Pb, 02.30.Ik

Introduction.—The S matrix is an object of central importance to quantum field theories. Unfortunately, it is extremely challenging to compute it because we typically have to rely on perturbative methods. Symmetries are extremely helpful in constraining the result. A couple of years ago it was noticed that loop scattering amplitudes in maximally supersymmetric Yang-Mills ($\mathcal{N}=4$ SYM) theory in the planar limit are simpler than one might expect [1,2] based on the known symmetries such as superconformal symmetry. This simplicity was traced to a hidden superconformal symmetry of dual Feynman graphs [3–5]. These observations have sparked enormous progress in the determination of the planar S matrix in this model, see [6,7] for reviews.

The appearance of dual symmetries is well understood from the point of view of the AdS/CFT dual string theory where ordinary and dual symmetries are exchanged by a T-self-duality [8–10]. The closure of these two sets of symmetries forms an infinite-dimensional algebra [11] known from the context of planar integrability [12]. Indeed, it was shown that the planar tree-level S matrix in $\mathcal{N}=4$ SYM is invariant under the Yangian $Y(\mathfrak{psu}(2,2|4))$ [13] for the superconformal algebra $\mathfrak{psu}(2,2|4)$.

In this Letter we find evidence for an additional symmetry of planar scattering amplitudes. It is not part of the above Yangian, but it fits nicely into its structure: It is the level-one recurrence $\tilde{\mathfrak{B}}$ of the $\mathfrak{u}(1)$ outer automorphism $\mathfrak{B}$ of $\mathfrak{psu}(2,2|4)$. The automorphism, sometimes called "bonus symmetry" [14], counts the helicity of particle states. We will refer to it as the hypercharge. It is clear that helicity is generally not conserved in scattering amplitudes, but here we will argue that the level-one hypercharge $\tilde{\mathfrak{B}}$ is indeed a proper symmetry.

An analogous observation has been made in the context of the world sheet S matrix [15] for planar $\mathcal{N}=4$ SYM. This S matrix is based on an extension of $\mathfrak{psu}(2|2)$ which also possesses a $\mathfrak{u}(1)$ automorphism. It was shown to be exactly symmetric under the Yangian level-one automorphism [16,17], sometimes called "secret symmetry," while there seem to be obstacles for the other levels.

In the following, we shall present evidence in favor of the Yangian level-one hypercharge $\tilde{\mathfrak{B}}$ being a symmetry of the planar S matrix: as a first check we show compatibility with the cyclic nature of color-ordered amplitudes. Next we confirm explicitly that tree-level MHV (maximally helicity violating) amplitudes are invariant. We will also check that the Grassmannian formula for leading singularities [18] respects our symmetry, and hence symmetry extends to all tree amplitudes at least. Finally, we show that our symmetry becomes exact in a distributional sense when appropriate correction terms are added.

Setup.—In the planar limit the S matrix is described by color-ordered scattering amplitudes $\mathcal{A}_n$. In $\mathcal{N}=4$ SYM the particle momentum, flavor, and helicity are conveniently encoded by a spinor-helicity superspace: the amplitude $\mathcal{A}_n$ is a function of the $\Lambda_k=(\lambda_k,\tilde{\lambda}_k,\eta_k)$, $k=1,\ldots,n$. The complex conjugate spinors $\lambda_k,\tilde{\lambda}_k\in\mathbb{C}^2$ describe a real massless momentum $p_k=\lambda_k\tilde{\lambda}_k$. Likewise, $\eta_k\in\mathbb{C}^{0|4}$ is a Grassmann variable to encode flavor.

Free superconformal symmetries $\mathfrak{J}^A$ are represented on particles by suitable differential operators $\mathfrak{J}^A_k$ acting on Λ_k [19]. As usual they act on an amplitude $\mathcal{A}_n$ as a sum over all external particles. Conversely, the Yangian symmetries $\tilde{\mathfrak{J}}^A$ act on pairs of external particles as follows:

$$\mathfrak{J}^A=\sum_{i=1}^{n}\mathfrak{J}^A_i,\qquad \tilde{\mathfrak{J}}^A=f^A_{BC}\sum_{j<k=1}^{n}\mathfrak{J}^B_j\mathfrak{J}^C_k. \tag{1}$$

Here f denotes the $\mathfrak{psu}(2,2|4)$ structure constants. When promoting them to $\mathfrak{u}(2,2|4)$ we find the free action of the Yangian level-one hypercharge $\tilde{\mathfrak{B}}$

$$\tilde{\mathfrak{B}}=\sum_{k=1}^{n-1}\sum_{j=k+1}^{n}(\mathfrak{Q}^{\alpha b}_k\mathfrak{S}_{j,\alpha b}-\bar{\mathfrak{Q}}^{\dot\alpha}_{k,b}\bar{\mathfrak{S}}^{b}_{j,\dot\alpha}-\mathfrak{Q}^{\alpha b}_j\mathfrak{S}_{k,\alpha b}+\bar{\mathfrak{Q}}^{\dot\alpha}_{j,b}\bar{\mathfrak{S}}^{b}_{k,\dot\alpha}). \tag{2}$$

Here $\mathfrak{Q}$ and $\mathfrak{S}$ denote the superconformal translations and boosts, respectively. Gladly, the broken hypercharge $\mathfrak{B}=\sum_i\eta^A_i\partial/\partial\eta^A_i$ does not appear in $\tilde{\mathfrak{B}}$. In the following we will argue that the bonus Yangian generator $\tilde{\mathfrak{B}}$ defined in (2) is a symmetry of the planar S matrix.

Cyclicity.—Color-ordered amplitudes are invariant under cyclic shifts of the external particles. Symmetries have to respect this property. The generators $\tilde{\mathfrak{J}}^A$ in (1) are cyclic, whereas the $\tilde{\mathfrak{J}}^A$ typically map cyclic functions to noncyclic ones. This can be seen by shifting the summation range in (1) by one unit [13]

$$\tilde{\mathfrak{J}}^A_{(2,n+1)} - \tilde{\mathfrak{J}}^A_{(1,n)} = f^A_{BC}\mathfrak{J}^B_1\mathfrak{J}^C + f^A_{BC}f^{BC}_D\mathfrak{J}^D_1. \quad (3)$$

Luckily, for scattering amplitudes the right-hand side is zero: the first term vanishes because $\mathfrak{J}^A$ is a symmetry, and the second one because the dual Coxeter number of $\mathfrak{psu}(2,2|4)$ is zero. Hence the Yangian generators are cyclic.

For the proposed bonus Yangian symmetry $\tilde{\mathfrak{B}}$ the situation is slightly different: the first term vanishes as before due to superconformal symmetry. In $\mathfrak{u}(2,2|4)$ the combination $f^A_{BC}f^{BC}_D$ is proportional to $\delta^A_{\mathfrak{B}}\delta^{\mathfrak{C}}_D$, and we obtain an additional $\mathfrak{C}_1$. This vanishes because the central charge of all individual particles is zero, and $\tilde{\mathfrak{B}}$ is indeed cyclic.

Invariance of MHV amplitudes.—First act with $\tilde{\mathfrak{B}}$ in (2) on MHV amplitudes [20,21]

$$\mathcal{A}_{n,2} = \frac{\delta^4(P)\delta^8(Q)}{\prod_j\langle j, j+1\rangle}, \quad P = \sum_j \lambda_j\tilde{\lambda}_j, \quad Q = \sum_j \lambda_j\eta_j, \quad (4)$$

with the spinor product $\langle j, k\rangle := \varepsilon_{\alpha\beta}\lambda^\alpha_j\lambda^\beta_k$. The fermionic derivatives in $\mathfrak{S}$, $\bar{\mathfrak{Q}}$ act only on $\delta^8(Q)$, and we obtain

$$\mathfrak{Q}^{\alpha b}_j\mathfrak{S}_{k,\alpha b}\mathcal{A}_{n,2} = \lambda^\alpha_j\eta^b_j\frac{\partial}{\partial\lambda^\alpha_k}\lambda^\beta_k\frac{\partial\delta^8(Q)}{\partial Q^{\beta b}}\frac{\delta^4(P)}{\prod_i\langle i, i+1\rangle}, \quad (5)$$

$$\bar{\mathfrak{Q}}^{\dot\alpha}_{k,b}\bar{\mathfrak{S}}^b_{j,\dot\alpha}\mathcal{A}_{n,2} = -\eta^b_j\frac{\partial}{\partial\tilde{\lambda}^{\dot\alpha}_j}\tilde{\lambda}^{\dot\alpha}_k\lambda^\beta_k\frac{\partial\delta^8(Q)}{\partial Q^{\beta b}}\frac{\delta^4(P)}{\prod_i\langle i, i+1\rangle}. \quad (6)$$

The bosonic derivative in (5) acts on the λ^β_k, both delta functions and the spinor brackets in the denominator. The action on $\delta^4(P)$ produces

$$\frac{\lambda^\alpha_j\eta^b_j\lambda^\beta_k\tilde{\lambda}^{\dot\alpha}_k}{\prod_i\langle i, i+1\rangle}\frac{\partial\delta^8(Q)}{\partial Q^{b\beta}}\frac{\partial\delta^4(P)}{\partial P^{\alpha\dot\alpha}}, \quad (7)$$

which cancels precisely the contribution from (6). The contribution from the action on $\delta^{(8)}(Q)$ is proportional to $\lambda^\alpha_j\eta^a_j\lambda^\beta_k\eta^b_k\partial^2\delta^8(Q)/\partial Q^{a\beta}\partial Q^{b\alpha}$. This expression is symmetric in j and k and vanishes due to the antisymmetry of (2). Next consider the contribution originating from the derivative acting on the spinor brackets. Combining the contributions from $\partial/\partial\lambda_k$ and $\partial/\partial\lambda_{k+1}$ (after a shift $k\to k+1$) acting both on the same $\langle k, k+1\rangle$ we obtain

$$-\lambda^\alpha_j\eta^a_j\frac{\partial\delta^8(Q)}{\partial Q^{\alpha a}}\frac{\delta^4(P)}{\prod_i\langle i, i+1\rangle}. \quad (8)$$

This term cancels identically with the derivative acting on λ^β_k in (5). The shift $k\to k+1$ leaves behind two boundary terms in the sum (2). Here we complete the sum $\sum_j\lambda_j\eta_j = Q$ and move it past the derivative acting on $\delta^8(Q)$. A careful calculation shows that all remaining terms cancel. Thus we find that $\tilde{\mathfrak{B}}$ leaves MHV amplitudes invariant, $\tilde{\mathfrak{B}}\mathcal{A}_{n,2} = 0$.

Invariance of the Grassmannian integral.—We complete the proof of invariance of tree amplitudes under $\tilde{\mathfrak{B}}$ using the Grassmannian integral formula [18] for leading singularities $\mathcal{L}_{n,k}$ in N^{k-2}MHV amplitudes $\mathcal{A}_{n,k}$ with $2 < k \leq n-2$

$$\mathcal{L}_{n,k} \simeq \int\frac{d^{k\times n}t\prod_{a=1}^k\delta_a}{\mathcal{M}_1(t)\cdots\mathcal{M}_n(t)}, \quad \delta_a = \delta^{4|4}\Big(\sum_{j=1}^n t_{aj}\mathcal{Z}_j\Big). \quad (9)$$

Here t is a $k\times n$ matrix, and $\mathcal{M}_a$ represent its minors of k consecutive rows starting at a. The particle momenta are encoded using supertwistors $\mathcal{Z}^{\mathcal{A}} = (\tilde{\mu}^\alpha, \tilde{\lambda}^{\dot\alpha}, \eta^a)$ with $\tilde{\mu}$ the Fourier conjugate to λ [19] (the calculation using momentum twistors [22] is virtually the same). The $\mathfrak{u}(2,2|4)$ algebra is now represented on particles by linear differential operators $\mathfrak{J}^{\mathcal{A}}_{\mathcal{B}} = (-1)^{\mathcal{B}}\mathcal{Z}^{\mathcal{A}}\partial_{\mathcal{B}}$. The corresponding Yangian generators take the form

$$\tilde{\mathfrak{J}}^{\mathcal{A}}_{\mathcal{B}} = \tilde{\mathfrak{J}}^{\mathcal{A}}_{<,\mathcal{B}} - \tilde{\mathfrak{J}}^{\mathcal{A}}_{>,\mathcal{B}}, \qquad \tilde{\mathfrak{J}}^{\mathcal{A}}_{\lessgtr,\mathcal{B}} = \sum_{j\lessgtr k=1}^n \mathfrak{J}^{\mathcal{A}}_{j,\mathcal{C}}\mathfrak{J}^{\mathcal{C}}_{k,\mathcal{B}}. \quad (10)$$

In this form our generator reads $\tilde{\mathfrak{B}} = \tilde{\mathfrak{J}}^{\mathcal{A}}_{\mathcal{A}}$.

We shall now show that both contributions $\tilde{\mathfrak{B}}_< \simeq \tilde{\mathfrak{B}}_> \simeq 4k(k-1)$ on $\mathcal{L}_{n,k}$, and hence $\tilde{\mathfrak{B}}$ annihilates the Grassmannian integral. Our proof and notation follows along the lines of [23], where the calculation for all previously known Yangian operators can be found in detail. We apply $\tilde{\mathfrak{B}}_<$ to $\mathcal{L}_{n,k}$ and obtain

$$\sum_{b=1}^k\int\frac{d^{k\times n}t}{\mathcal{M}_1\cdots\mathcal{M}_n}(-1)^{\mathcal{A}}[\mathcal{O}^{\mathcal{A}}_b - \mathcal{V}^{\mathcal{A}}_b](\partial_{\mathcal{A}}\delta_b)\prod_{a\neq b}\delta_a. \quad (11)$$

We have defined $\mathcal{O}^{\mathcal{A}}_b := \sum_{a,i<j}\mathcal{Z}^{\mathcal{A}}_i t_{ai}(\partial/\partial t_{aj})t_{bj}$ and $\mathcal{V}^{\mathcal{A}}_b := \sum_{i<j}\mathcal{Z}^{\mathcal{A}}_i t_{bi}$. Now we commute the operator $\mathcal{O}^{\mathcal{A}}_b$ past the minors $\mathcal{M}_p$ in the denominator. At this point it is important to be very careful as to not overlook the contributions that arise from the supertrace over the index $\mathcal{A}$, cf. footnote 9 in [23]: specifically due to the wrapping of the minors $\mathcal{M}_p$ around the end of the $k\times n$ matrix t_{bj} it is necessary to make a distinction between the two cases $p\leq n-k+1$ and $p > n-k+1$. In the first case the supertrace has no impact on the calculation. In the latter case, however, it is inevitable to use the constraint from the delta functions δ_a twice. For the delta function bearing the derivative $\partial_{\mathcal{A}}\delta_b$ the supertrace leaves an extra term proportional to the Grassmannian integral. The result of this operation is given by

$$(-1)^{\mathcal{A}} \sum_{b=1}^{k} \sum_{p=n-k+2}^{n} \sum_{s=p}^{n} \sum_{i=1}^{s-1} \frac{1}{\mathcal{M}_p} Z_i^{\mathcal{A}} t_{bs} \mathcal{M}_p^{s\to i}(\partial_{\mathcal{A}} \delta_b)$$
$$= - \sum_{s\geq p=n-k+2}^{n} \Big[8\delta_b + (-1)^{\mathcal{A}} \sum_{b=1}^{k} Z_s^{\mathcal{A}} t_{bs}(\partial_{\mathcal{A}}\delta_b) \Big], \quad (12)$$

after adding and subtracting the terms missing to make the sum over s range from 1 to n and then performing the partial integration of the derivative $\partial_{\mathcal{A}}$. The form of the second part on the right-hand side follows from the antisymmetry of the minors which singles out the terms with $i = s$. The two sums in the first term evaluate straightforwardly to $-4k(k-1)$. Repeating this procedure for the second term on the right-hand side of (12) yields a factor of $8k(k-1)$, such that in the end one is left with $4k(k-1)\mathcal{L}_{n,k}$. The contributions from $\tilde{\mathfrak{B}}_<$ and $\tilde{\mathfrak{B}}_>$ cancel each other leaving only a total derivative under the integral as in the proof of [23]. This confirms that $\tilde{\mathfrak{B}}$ is a symmetry of all leading singularities and, in particular, of tree-level amplitudes of $\mathcal{N} = 4$ SYM.

Distributional contributions.—Because of the holomorphic anomaly $(\partial/\partial\bar{\lambda}^{\dot{\alpha}})\langle\lambda,\mu\rangle^{-1} = 2\pi\varepsilon_{\dot{\alpha}\dot{\beta}}\bar{\mu}^{\dot{\beta}}\delta^2(\langle\lambda,\mu\rangle)$ the above derivations disregard certain distributional contributions which at first sight violate the exactness of the symmetry. In [24] it was shown that in the case of superconformal boosts $\mathfrak{S}$, $\bar{\mathfrak{S}}$ the representation can be corrected to restore the symmetry. The correction terms are operators $\mathfrak{S}^+$, $\bar{\mathfrak{S}}^+$ which act on an amplitude with $n-1$ legs and return an amplitude with n legs. The statement of exact invariance then takes the form $\mathfrak{S}\mathcal{A}_n + \mathfrak{S}^+\mathcal{A}_{n-1} = 0$. As our generator $\tilde{\mathfrak{B}}$ contains the superconformal boosts we will have to correct it by a suitable length-changing deformation $\tilde{\mathfrak{B}}^+$ such that

$$\tilde{\mathfrak{B}}\mathcal{A}_n + \tilde{\mathfrak{B}}^+\mathcal{A}_{n-1} = 0. \quad (13)$$

Before we consider the correction, let us briefly discuss how to work with length-changing operators. The correction $\mathfrak{S}^+$ acts on an $(n-1)$-particle function and generates an n-particle function. We define the action on the first leg via a three-vertex S^+ [25]

$$(\mathfrak{S}_1^+ F_{n-1}) := \int d^{4|4}\Lambda S^+(1,2,\bar{\Lambda}) F_{n-1}(\Lambda, 3, \ldots, n). \quad (14)$$

Note that it shifts all the legs $2, \ldots n-1$ of F_{n-1} by one index to $3, \ldots, n$. We then use the cyclic shift operator $(\mathcal{U}_n F_n)(1,\ldots,n) := F_n(2,\ldots,n,1)$ to bring the correction term into all possible places [26]

$$\mathfrak{S}^+ = \sum_{k=1}^{n} \mathfrak{S}_k^+, \qquad \mathfrak{S}_k^+ := \mathcal{U}_n^{k-1}\mathfrak{S}_1^+\mathcal{U}_{n-1}^{1-k}. \quad (15)$$

For our new symmetry generator $\tilde{\mathfrak{B}}$ we propose the following correction term $\tilde{\mathfrak{B}}^+$:

$$\tilde{\mathfrak{B}}^+ = \sum_{k=1}^{n-1}\sum_{j=k+1}^{n} (\mathfrak{Q}_k^{\alpha b}\mathfrak{S}_{j,\alpha b}^+ - \bar{\mathfrak{Q}}_{k,b}^{\dot{\alpha}}\bar{\mathfrak{S}}_{j,\dot{\alpha}}^{+,b}$$
$$- \mathfrak{Q}_j^{\alpha b}\mathfrak{S}_{k-1,\alpha b}^+ + \bar{\mathfrak{Q}}_{j,b}^{\dot{\alpha}}\bar{\mathfrak{S}}_{k-1,\dot{\alpha}}^{+,b}). \quad (16)$$

Note the shift of argument for $\mathfrak{S}^+$ as compared to (2) when $\mathfrak{Q}$ acts further to the right.

As a first check we consider cyclicity of $\tilde{\mathfrak{B}} + \tilde{\mathfrak{B}}^+$ (supposing we act on cyclic functions)

$$(\mathcal{U}_n - 1)(\tilde{\mathfrak{B}} + \tilde{\mathfrak{B}}^+) = -2\mathfrak{Q}_1^{\alpha b}(\mathfrak{S}_{\alpha b} + \mathfrak{S}_{\alpha b}^+)$$
$$+ 2\bar{\mathfrak{Q}}_{1,b}^{\dot{\alpha}}(\bar{\mathfrak{S}}_{\dot{\alpha}}^b + \bar{\mathfrak{S}}_{\dot{\alpha}}^{+,b})$$
$$+ \mathfrak{Q}^{\alpha B}(2\mathfrak{S}_{1,\alpha b} + \mathfrak{S}_{0,\alpha b}^+ + \mathfrak{S}_{1,\alpha b}^+)$$
$$- \bar{\mathfrak{Q}}_b^{\dot{\alpha}}(2\bar{\mathfrak{S}}_{1,\dot{\alpha}}^b + \bar{\mathfrak{S}}_{1,\dot{\alpha}}^{+,B} + \bar{\mathfrak{S}}_{0,\dot{\alpha}}^{+,b}). \quad (17)$$

The $\mathfrak{Q}$'s anticommute exactly with the $\mathfrak{S}_k^+$'s [24], therefore the action of $\tilde{\mathfrak{B}} + \tilde{\mathfrak{B}}^+$ is cyclic. Interestingly, only the combination of $\tilde{\mathfrak{B}}$ and $\tilde{\mathfrak{B}}^+$ is cyclic because only the combination $\mathfrak{S} + \mathfrak{S}^+$ annihilates amplitudes exactly.

More importantly, we can show exact invariance of MHV amplitudes. To show (13) we note the action of $\bar{\mathfrak{S}}^+$

$$\bar{\mathfrak{S}}_{k,\dot{\alpha}}^{+,b}\mathcal{A}_{n-1,2} = 2\pi\varepsilon_{\dot{\alpha}\dot{\beta}}(\bar{\lambda}_k^{\dot{\beta}}\eta_{k+1}^b - \bar{\lambda}_{k+1}^{\dot{\beta}}\eta_k^b)$$
$$\cdot \delta^2(\langle k, k+1\rangle)\frac{\delta^8(Q)\delta^4(P)}{\prod_{i\neq k}\langle i, i+1\rangle}. \quad (18)$$

By construction almost all distributional terms cancel. Only at the boundary there are some residual terms for which we need some identities to show full cancellation

$$0 = \varepsilon_{\dot{\alpha}\dot{\beta}}\bar{\mathfrak{Q}}_{1,b}^{\dot{\alpha}}\bar{\lambda}_1^{\dot{\beta}} = \varepsilon_{\dot{\alpha}\dot{\beta}}\bar{\mathfrak{Q}}_{1,b}^{\dot{\alpha}}\bar{\lambda}_2^{\dot{\beta}}\eta_1^b\delta^2(\langle 1,2\rangle)$$
$$= \bar{\mathfrak{Q}}_b^{\dot{\alpha}}\delta^8(Q)\delta^4(P). \quad (19)$$

Conclusions and outlook.—In view of the conjectured integrability for the planar S matrix of $\mathcal{N} = 4$ SYM, and its many useful applications, it is extremely important to understand the underlying symmetries. In this Letter, we have proposed that there exists an exact symmetry besides the established Yangian algebra $Y(\mathfrak{psu}(2,2|4))$. This Yangian-like symmetry generator $\tilde{\mathfrak{B}}$ is the level-one recurrence of the hypercharge $\mathfrak{B}$, both of which are included in the bigger algebra $Y(\mathfrak{u}(2,2|4))$. Now, curiously, the novel $\tilde{\mathfrak{B}}$ appears to be a symmetry whereas $\mathfrak{B}$ clearly is none. This leads to an intriguing structure of the symmetry algebra somewhere in between $Y(\mathfrak{psu}(2,2|4))$ and $Y(\mathfrak{u}(2,2|4))$.

We have shown explicitly that the bonus Yangian symmetry $\tilde{\mathfrak{B}}$ is a symmetry of all tree-level amplitudes, and argued that the symmetry is exact in a distributional sense, at least for MHV amplitudes. Cyclicity of color-ordered amplitudes is respected (3) and (17). All this, in conjunction with the invariance of the Grassmanian integral, leads to the conclusion that $\tilde{\mathfrak{B}}$ stands a good chance of being a symmetry of loop amplitudes [27]. Similarly the question arises whether $\tilde{\mathfrak{B}}$ is a symmetry of the (bulk) higher-loop spin chain Hamiltonian for planar anomalous dimensions of local operators, cf. the reviews [12,28].

Notably, the new symmetry is stronger than the dual symmetries. Together with the ordinary superconformal symmetries we can generate all previously known symmetries of the S matrix including the dual superconformal ones (this also holds when the correction term $\tilde{\mathfrak{B}}^+$ is considered) via

$$[\tilde{\mathfrak{B}}, \mathfrak{Q}] = +\tilde{\mathfrak{Q}}, \qquad [\tilde{\mathfrak{B}}, \mathfrak{S}] = -\tilde{\mathfrak{S}}, \qquad \ldots. \tag{20}$$

Conversely, the ordinary and dual superconformal symmetries only close onto the Yangian $Y(\mathfrak{psu}(2,2|4))$. As an outer automorphism our symmetry can never be generated in this fashion. Therefore one might wonder if $\tilde{\mathfrak{B}}$ actually yields stronger constraints for the S matrix than the dual symmetries: abstractly this is to be expected, but potentially the S matrix is special and invariance under $\tilde{\mathfrak{B}}$ is automatic, cf. [29]. Invariance of the proposed all-loop integrand [30] in fact follows from invariance of the Grassmannian integral by construction.

It would also be desirable to shed some light on the (geometric) transformation induced by $\tilde{\mathfrak{B}}$ which is at the first level of the Yangian in both the original and dual picture of the S matrix, i.e., it is simple in neither picture.

To finish, we comment on scattering amplitudes of $\mathcal{N} = 6$ super–Chern-Simons theory, which enjoy a similar Yangian symmetry [31]. Its Yangian $Y(\mathfrak{osp}(6|4))$ does not admit an outer automorphism, however, the action of the generator $\tilde{\mathfrak{R}}$ is somewhat reminiscent of our bonus Yangian symmetry $\tilde{\mathfrak{B}}$.

We would like to thank L. Ferro, T. Matsumoto, T. McLoughlin, and J. Plefka for useful discussions. The work of N. B. is supported in part by the German-Israeli Foundation (GIF).

*nbeisert@aei.mpg.de
†buws2@aei.mpg.de

[1] C. Anastasiou, Z. Bern, L. J. Dixon, and D. A. Kosower, Phys. Rev. Lett. **91**, 251602 (2003).
[2] Z. Bern, L. J. Dixon, and V. A. Smirnov, Phys. Rev. D **72**, 085001 (2005).
[3] J. M. Drummond, J. Henn, V. A. Smirnov, and E. Sokatchev, J. High Energy Phys. 01 (2007) 064.
[4] J. M. Drummond, J. Henn, G. P. Korchemsky, and E. Sokatchev, Nucl. Phys. **B828**, 317 (2010).
[5] A. Brandhuber, P. Heslop, and G. Travaglini, Phys. Rev. D **78**, 125005 (2008).
[6] L. F. Alday and R. Roiban, Phys. Rep. **468**, 153 (2008).
[7] J. M. Drummond, arXiv:1012.4002.
[8] L. F. Alday and J. M. Maldacena, J. High Energy Phys. 06 (2007) 064.
[9] N. Beisert, R. Ricci, A. A. Tseytlin, and M. Wolf, Phys. Rev. D **78**, 126004 (2008).
[10] N. Berkovits and J. Maldacena, J. High Energy Phys. 09 (2008) 062.
[11] N. Beisert, Fortschr. Phys. **57**, 329 (2009).
[12] N. Beisert *et al.*, arXiv:1012.3982.
[13] J. M. Drummond, J. M. Henn, and J. Plefka, J. High Energy Phys. 05 (2009) 046.
[14] K. A. Intriligator, Nucl. Phys. **B551**, 575 (1999).
[15] N. Beisert, Adv. Theor. Math. Phys. **12**, 945 (2008).
[16] T. Matsumoto, S. Moriyama, and A. Torrielli, J. High Energy Phys. 09 (2007) 099.
[17] N. Beisert and F. Spill, Commun. Math. Phys. **285**, 537 (2008).
[18] N. Arkani-Hamed, F. Cachazo, C. Cheung, and J. Kaplan, J. High Energy Phys. 03 (2010) 020.
[19] E. Witten, Commun. Math. Phys. **252**, 189 (2004).
[20] S. J. Parke and T. R. Taylor, Phys. Rev. Lett. **56**, 2459 (1986).
[21] F. A. Berends and W. T. Giele, Nucl. Phys. **B306**, 759 (1988).
[22] L. Mason and D. Skinner, J. High Energy Phys. 11 (2009) 045.
[23] J. M. Drummond and L. Ferro, J. High Energy Phys. 07 (2010) 027.
[24] T. Bargheer, N. Beisert, W. Galleas, F. Loebbert, and T. McLoughlin, J. High Energy Phys. 11 (2009) 056.
[25] N. Beisert, J. Henn, T. McLoughlin, and J. Plefka, J. High Energy Phys. 04 (2010) 085.
[26] Note that the shift operators $\mathcal{U}_n$ and $\mathcal{U}_{n-1}$ act on two different spaces. Thus $\mathfrak{S}_k^+$ is not periodic: $\mathfrak{S}_{k+n}^+ = \mathfrak{S}_k^+ \mathcal{U}_{n-1}^{-1}$. In physical situations we act only on cyclic states where $\mathcal{U}_{n-1} \simeq 1$ such that $\mathfrak{S}^+$ preserves cyclicity.
[27] Loop integrals break invariance of the S matrix, yet in a controllable fashion. We expect $\tilde{\mathfrak{B}}$ to behave like all previously known superconformal and Yangian generators.
[28] N. Beisert, Phys. Rep. **405**, 1 (2004).
[29] J. M. Drummond and L. Ferro, J. High Energy Phys. 12 (2010) 010.
[30] N. Arkani-Hamed, J. L. Bourjaily, F. Cachazo, S. Caron-Huot, and J. Trnka, J. High Energy Phys. 01 (2011) 041.
[31] T. Bargheer, F. Loebbert, and C. Meneghelli, Phys. Rev. D **82**, 045016 (2010).