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# On the fermionic T-duality of the $AdS_4 \times \mathbb{CP}^3$ sigma-model

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Abstract: In this note we consider a fermionic T-duality of the coset realization of the type IIA sigma-model on $AdS_4 \times \mathbb{CP}^3$ with respect to the three flat directions in $AdS_4$, six of the fermionic coordinates and three of the $\mathbb{CP}^3$ directions. We show that the Buscher procedure fails as it leads to a singular transformation and discuss the result and its implications.

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## Contents



## 1 Introduction and summary

Since the $\mathcal{N} = 6$ superconformal Chern-Simons theory with matter was proposed by ABJM [1] as a dual to M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$, which reduces in a certain limit to the type IIA superstring on $AdS_4 \times \mathbb{C}\mathrm{P}^3$, much work has been devoted to understanding the properties of the ABJM field theory.

Several tree-level scattering amplitudes of the ABJM theory were computed [2] and were shown to possess a Yangian symmetry, which includes the non-local charges and the dual superconformal symmetry [3]. Some light-like polygonal Wilson loops in the ABJM theory were computed in [4] and hinted that the ABJM theory may have a scattering amplitudes/Wilson loop duality, which would further support the case in favor of the existence of dual superconformal symmetry. Additionally, a contour integral reproducing the known tree-level amplitudes has been recently proposed and was shown to have a Yangian symmetry [5]. Furthermore, a differential representation of a dual superconformal symmetry at tree-level has been constructed [6]. This representation involves variables dual to the ones parameterizing part of the R-symmetry in addition to the ones dual to the bosonic and fermionic momenta.

The corresponding findings in $\mathcal{N} = 4$ SYM in four dimensions were explained from the point of view of string theory on $AdS_5 \times S^5$ by a combination of bosonic and fermionic T-dualities, which is exact at the string tree-level [7, 8] (see [9] for a short review). Hence, it is interesting to see whether that is also the case for type IIA strings on $AdS_4 \times \mathbb{C}\mathrm{P}^3$. Previously, it was found that the sigma-model for $AdS_4 \times \mathbb{C}\mathrm{P}^3$, realized as the coset $\mathrm{OSp}(6|4)/(\mathrm{SO}(2,1) \times \mathrm{U}(3))$ constructed in [10, 11], was not self-dual under T-duality involving both three directions in $AdS_4$ and six fermionic coordinates [12, 13]. In fact, one could not perform a fermionic T-duality in six fermionic isometries which together with the dualized bosonic ones form an Abelian subgroup of the whole isometry group.

In this note, in light of a suggestion that T-dualizing three isometries of $\mathbb{C}\mathrm{P}^3$ is also required [3] and the new evidence [5, 6] from the field theory, we consider the fermionic T-duality along the three flat $AdS_4$ coordinates, three complex Killing vectors in $\mathbb{C}\mathrm{P}^3$ (each one of real dimension one) as well as six of the fermionic coordinates, whose corresponding tangent-space vectors generate an Abelian subgroup of the isometry group. We show that as in the case of dualizing just in $AdS_4$ and the fermions, the Buscher procedure fails as it leads to a singular transformation [12].

The outline of this note is as follows: in section 2 we apply the Buscher procedure for T-duality to the OSp(6|4)/(SO(2,1) × U(3)) Green-Schwarz sigma-model describing type IIA strings on $AdS_4 \times \mathbb{C}\mathrm{P}^3$ in a certain partial gauge-fixing and show that it fails. In section 3 we discuss the implications of the result. The osp(6|4) algebra is given in appendix A.

## 2 T-dualizing $AdS_4 \times \mathbb{C}\mathrm{P}^3$

We attempt to T-dualize $AdS_4 \times \mathbb{C}\mathrm{P}^3$ along the directions corresponding to $P_a$, $Q_{l\alpha}$, $R_{kl}$, which form an Abelian subalgebra of the isometry group.

We assume that $\kappa$-symmetry can be partially gauge-fixed to set the six coordinates corresponding to $\hat{S}^l_\alpha$ to zero and choose the coset representative

$$g = e^{x^a P_a + \theta^{l\alpha} Q_{l\alpha} + y^{kl} R_{kl}} e^B \,, \quad e^B = e^{\hat{\theta}^\alpha_l \hat{Q}^l_\alpha + \xi^{l\alpha} S_{l\alpha}} y^D e^{\hat{y}_{kl} \hat{R}^{kl}} \,, \tag{2.1}$$

where the indices $a = 0, 1, 2$ run over the flat directions of $AdS_4$, $\alpha = 1, 2$ are $AdS_4$ spinor indices and $l = 1, 2, 3$ are U(3) fundamental representation indices (see appendix A for further details). The Maurer-Cartan one-form is

$$K = J + j \,, \quad J = e^{-B}(dx^a P_a + d\theta^{l\alpha} Q_{l\alpha} + dy^{kl} R_{kl}) e^B \,, \quad j = e^{-B} de^B \,. \tag{2.2}$$

Examining the algebra, one finds that the current $J$ takes values in the space spanned by $\{P_a, Q_{l\alpha}, R_{kl}, \hat{Q}^l_\alpha, \lambda_k{}^l, \hat{R}^{kl}\}$, while $j$ is valued in span$\{\hat{Q}^l_\alpha, S_{l\alpha}, \hat{S}^l_\alpha, D, M_{ab}, \lambda_k{}^l, \hat{R}^{kl}\}$.

Denoting the decomposition of $K$ into the $\mathbb{Z}_4$-invariant subspaces by $K_i \in \mathcal{H}_i$, the Green-Schwarz action takes the form

$$\begin{aligned} S = \frac{R^2}{4\pi\alpha'} \int d^2z \Big\{ & -\frac{1}{2}\eta_{ab} J_{P_a} \bar{J}_{P_b} - j_D \bar{j}_D - 2J_{R_{kl}}(\bar{J}_{\hat{R}^{kl}} + \bar{j}_{\hat{R}^{kl}}) - 2\bar{J}_{R_{kl}}(J_{\hat{R}^{kl}} + j_{\hat{R}^{kl}}) - \\ & - \frac{i}{2} C_{\alpha\beta} \Big[ J_{Q_{l\alpha}}(\bar{J}_{\hat{Q}^l_\beta} + \bar{j}_{\hat{Q}^l_\beta}) - (J_{\hat{Q}^l_\alpha} + j_{\hat{Q}^l_\alpha}) \bar{J}_{Q_{l\beta}} - j_{S_{l\alpha}} \bar{j}_{\hat{S}^l_\beta} + j_{\hat{S}^l_\alpha} \bar{j}_{S_{l\beta}} \Big] \Big\}. \end{aligned} \tag{2.3}$$

We attempt to T-dualize the action by using the Buscher procedure [14, 15] by introducing the new fields $A^a$, $A^{l\alpha}$, $A^{kl}$, $\bar{A}^a$, $\bar{A}^{l\alpha}$ and $\bar{A}^{kl}$ such that the current now reads

$$J = e^{-B}(A^a P_a + A^{l\alpha} Q_{l\alpha} + A^{kl} R_{kl}) e^B \,, \tag{2.4}$$

while $j$, which does not contain $x^a$, $\theta^{l\alpha}$ and $y^{kl}$, remains unmodified. In addition, the following Lagrange multiplier terms are added to the action:

$$S_{\mathrm{L}} = \frac{R^2}{4\pi\alpha'} \int d^2z \left[ \tilde{x}_a(\bar{\partial} A^a - \partial \bar{A}^a) + \tilde{\theta}_{l\alpha}(\bar{\partial} A^{l\alpha} - \partial \bar{A}^{l\alpha}) + \tilde{y}_{kl}(\bar{\partial} A^{kl} - \partial \bar{A}^{kl}) \right] , \tag{2.5}$$

where $\tilde{x}_a$, $\tilde{\theta}_{l\alpha}$ and $\tilde{y}_{kl}$ are Lagrange multipliers.



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The T-duality is performed by integrating out the gauge fields, whose equations of motion are

$$\begin{aligned}
0 = & -\frac{1}{2}\eta_{bc}[e^{-B}P_ae^B]_{P_b}J_{P_c} + \frac{i}{2}C_{\alpha\beta}\Big[[e^{-B}P_ae^B]_{Q_{l\alpha}}(J_{\hat{Q}^l_\beta} + j_{\hat{Q}^l_\beta}) - \\
& - [e^{-B}P_ae^B]_{\hat{Q}^l_\alpha}J_{Q_{l\beta}}\Big] - 2[e^{-B}P_ae^B]_{R_{kl}}(J_{\hat{R}^{kl}} + j_{\hat{R}^{kl}}) - 2[e^{-B}P_ae^B]_{\hat{R}^{kl}}J_{R_{kl}} + \partial\tilde{x}_a\,, \\
0 = & -\frac{1}{2}\eta_{bc}[e^{-B}Q_{l\alpha}e^B]_{P_b}J_{P_c} + \frac{i}{2}C_{\beta\gamma}\Big[[e^{-B}Q_{l\alpha}e^B]_{Q_{k\beta}}(J_{\hat{Q}^k_\gamma} + j_{\hat{Q}^k_\gamma}) - \\
& - [e^{-B}Q_{l\alpha}e^B]_{\hat{Q}^k_\beta}J_{Q_{k\gamma}}\Big] - 2[e^{-B}Q_{l\alpha}e^B]_{R_{pq}}(J_{\hat{R}^{pq}} + j_{\hat{R}^{pq}}) - 2[e^{-B}Q_{l\alpha}e^B]_{\hat{R}^{pq}}J_{R_{pq}} - \\
& - \partial\tilde{\theta}_{l\alpha}\,, \\
0 = & -\frac{1}{2}\eta_{bc}[e^{-B}R_{kl}e^B]_{P_b}J_{P_c} + \frac{i}{2}C_{\alpha\beta}\Big[[e^{-B}R_{kl}e^B]_{Q_{p\alpha}}(J_{\hat{Q}^p_\beta} + j_{\hat{Q}^p_\beta}) - \\
& - [e^{-B}R_{kl}e^B]_{\hat{Q}^p_\alpha}J_{Q_{p\beta}}\Big] - 2[e^{-B}R_{kl}e^B]_{R_{pq}}(J_{\hat{R}^{pq}} + j_{\hat{R}^{pq}}) - 2[e^{-B}R_{kl}e^B]_{\hat{R}^{pq}}J_{R_{pq}} + \\
& + \partial\tilde{y}_{kl}
\end{aligned} \tag{2.6}$$

for the holomorphic fields and

$$\begin{aligned}
0 = & -\frac{1}{2}\eta_{bc}[e^{-B}P_ae^B]_{P_b}\bar{J}_{P_c} - \frac{i}{2}C_{\alpha\beta}\Big[[e^{-B}P_ae^B]_{Q_{l\alpha}}(\bar{J}_{\hat{Q}^l_\beta} + \bar{j}_{\hat{Q}^l_\beta}) - [e^{-B}P_ae^B]_{\hat{Q}^l_\alpha}\bar{J}_{Q_{l\beta}}\Big] - \\
& - 2[e^{-B}P_ae^B]_{R_{kl}}(\bar{J}_{\hat{R}^{kl}} + \bar{j}_{\hat{R}^{kl}}) - 2[e^{-B}P_ae^B]_{\hat{R}^{kl}}\bar{J}_{R_{kl}} - \bar{\partial}\tilde{x}_a\,, \\
0 = & -\frac{1}{2}\eta_{bc}[e^{-B}Q_{l\alpha}e^B]_{P_b}\bar{J}_{P_c} - \frac{i}{2}C_{\beta\gamma}\Big[[e^{-B}Q_{l\alpha}e^B]_{Q_{k\beta}}(\bar{J}_{\hat{Q}^k_\gamma} + \bar{j}_{\hat{Q}^k_\gamma}) - \\
& - [e^{-B}Q_{l\alpha}e^B]_{\hat{Q}^k_\beta}\bar{J}_{Q_{k\gamma}}\Big] - 2[e^{-B}Q_{l\alpha}e^B]_{R_{pq}}(\bar{J}_{\hat{R}^{pq}} + \bar{j}_{\hat{R}^{pq}}) - 2[e^{-B}Q_{l\alpha}e^B]_{\hat{R}^{pq}}\bar{J}_{R_{pq}} + \\
& + \bar{\partial}\tilde{\theta}_{l\alpha}\,, \\
0 = & -\frac{1}{2}\eta_{bc}[e^{-B}R_{kl}e^B]_{P_b}\bar{J}_{P_c} - \frac{i}{2}C_{\alpha\beta}\Big[[e^{-B}R_{kl}e^B]_{Q_{p\alpha}}(\bar{J}_{\hat{Q}^p_\beta} + \bar{j}_{\hat{Q}^p_\beta}) - \\
& - [e^{-B}R_{kl}e^B]_{\hat{Q}^p_\alpha}\bar{J}_{Q_{p\beta}}\Big] - 2[e^{-B}R_{kl}e^B]_{R_{pq}}(\bar{J}_{\hat{R}^{pq}} + \bar{j}_{\hat{R}^{pq}}) - 2[e^{-B}R_{kl}e^B]_{\hat{R}^{pq}}\bar{J}_{R_{pq}} - \\
& - \bar{\partial}\tilde{y}_{kl}
\end{aligned} \tag{2.7}$$

for the anti-holomorphic ones. (The complexity of the equations arises from the fact that, unlike in the $AdS_5 \times S^5$ case, $J$ is valued in a space larger than the one that is actually dualized.)

For the purpose of solving these equations, the properties of the field-dependent group-theoretic factors must be understood. In particular, it should be checked whether the coefficients of the gauge fields have non-trivial kernels.

In order to do so, we resort to explicitly expressing the currents in terms of the coordinates. We denote $C \equiv \hat{\theta}^\alpha_l \hat{Q}^l_\alpha + \xi^{l\alpha}S_{l\alpha}$ and examine the commutators

$$\begin{aligned}
[P_a, C] &= -\frac{i}{\sqrt{2}}\gamma_{a\alpha}{}^{\beta}\xi^{l\alpha}Q_{l\beta} \equiv \Xi^{Pl\beta}_a Q_{l\beta}\,, \\
[Q_{l\beta}, C] &= \frac{1}{\sqrt{2}}(\gamma^a C)_{\beta\alpha}\hat{\theta}^\alpha_l P_a + \frac{1}{\sqrt{2}}C_{\beta\alpha}\xi^{k\alpha}R_{lk} \equiv \Theta^{Qa}_{l\beta}P_a + \Xi^{Qk}_\beta R_{lk} \equiv M_{l\beta}\,, \\
[R_{kl}, C] &= -\frac{i}{\sqrt{2}}(\hat{\theta}^\alpha_l\delta^p_k - \hat{\theta}^\alpha_k\delta^p_l)Q_{p\alpha} \equiv \Theta^{Rp\alpha}_{kl}Q_{p\alpha}\,.
\end{aligned} \tag{2.8}$$

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We further define

$$N_{l\alpha}{}^{k\beta} = \Theta^{Qa}_{l\alpha}\Xi^{Pk\beta}_{a} + \Xi^{Qp}_{\alpha}\Theta^{Rk\beta}_{pl} \tag{2.9}$$

and note that $[M_{l\alpha}, C] = N_{l\alpha}{}^{k\beta}Q_{k\beta}$ and $[Q_{l\alpha}, C] = M_{l\alpha}$. Using the formula $e^{-B}Ae^{B} = A + [A,B] + \frac{1}{2!}[[A,B],B] + \ldots$, we get

$$\begin{aligned}
&e^{-C}(dx^a P_a + d\theta^{l\alpha}Q_{l\alpha} + dy^{kl}R_{kl})e^{C} = dx^a P_a + dy^{kl}R_{kl} + \\
&+ (dx^a\Xi^{Pl\alpha}_{a} + dy^{pq}\Theta^{Rl\alpha}_{pq})\left[\left(\frac{\cosh\sqrt{N}-1}{N}\right)_{l\alpha}^{\;\;k\beta} M_{k\beta} + \left(\frac{\sinh\sqrt{N}}{\sqrt{N}}\right)_{l\alpha}^{\;\;k\beta} Q_{k\beta}\right] + \\
&+ d\theta^{l\alpha}\left[\left(\frac{\sinh\sqrt{N}}{\sqrt{N}}\right)_{l\alpha}^{\;\;k\beta} M_{k\beta} + (\cosh\sqrt{N})_{l\alpha}{}^{k\beta}Q_{k\beta}\right] .
\end{aligned} \tag{2.10}$$

Finally, conjugating with $y^{D}e^{\hat{y}_{kl}\hat{R}^{kl}}$ yields the current

$$\begin{aligned}
J = &\frac{dx^a}{y}P_a + dy^{kl}(R_{kl} + 2i\sqrt{2}\hat{y}_{kq}\lambda_l{}^q + 2\hat{y}_{kq}\hat{y}_{ln}\hat{R}^{qn}) + \\
&+ \left[(dx^a\Xi^{Pl\alpha}_{a} + dy^{pq}\Theta^{Rl\alpha}_{pq})\left(\frac{\cosh\sqrt{N}-1}{N}\right)_{l\alpha}^{\;\;k\beta} + d\theta^{l\alpha}\left(\frac{\sinh\sqrt{N}}{\sqrt{N}}\right)_{l\alpha}^{\;\;k\beta}\right]\times \\
&\times\left[\tilde{M}_{k\beta} + i\sqrt{2}\Xi^{Qm}_{\beta}(\hat{y}_{kq}\lambda_m{}^q - \hat{y}_{mq}\lambda_k{}^q) + \Xi^{Qr}_{\beta}(\hat{y}_{kq}\hat{y}_{rn} - \hat{y}_{rq}\hat{y}_{kn})\hat{R}^{qn}\right] + \\
&+ \frac{1}{y^{1/2}}\left[(dx^a\Xi^{Pl\alpha}_{a} + dy^{pq}\Theta^{Rl\alpha}_{pq})\left(\frac{\sinh\sqrt{N}}{\sqrt{N}}\right)_{l\alpha}^{\;\;k\beta} + d\theta^{l\alpha}(\cosh\sqrt{N})_{l\alpha}{}^{k\beta}\right]\times \\
&\times(Q_{k\beta} + i\sqrt{2}\hat{y}_{pk}\hat{Q}^{p}_{\beta}) ,
\end{aligned} \tag{2.11}$$

where $\tilde{M}_{k\beta} \equiv y^{-D}M_{k\beta}y^{D} = \frac{1}{y}\Theta^{Qa}_{l\alpha}P_a + \Xi^{Ql}_{\alpha}R_{kl}$.

Unfortunately, $j$ is even more complicated. However, before plunging into its computation in a closed form it is worthwhile to examine it to the lowest order in $\hat{\theta}^{\alpha}_{l}$ and $\xi^{l\alpha}$. Doing so yields,

$$j = \frac{d\hat{\theta}^{\alpha}_{l}}{y^{1/2}}\hat{Q}^{l}_{\alpha} + y^{1/2}d\xi^{l\alpha}S_{l\alpha} - i\sqrt{2}y^{1/2}\hat{y}_{kl}d\xi^{l\alpha}\hat{S}^{k}_{\alpha} + \frac{dy}{y}D + d\hat{y}_{pq}\hat{R}^{pq} + O(\hat{\theta}^{\alpha}_{l}, \xi^{l\alpha}) . \tag{2.12}$$

Having the currents, we can take a look at the action to lowest order in $\hat{\theta}^{\alpha}_{l}$ and $\xi^{l\alpha}$:

$$\begin{aligned}
S = &\frac{R^2}{4\pi\alpha'}\int d^2z\Big\{ -\frac{1}{2}\eta_{ab}\frac{\partial x^a\bar{\partial}x^b}{y^2} - \frac{\partial y\bar{\partial}y}{y^2} - 2\partial y^{kl}(2\hat{y}_{pk}\hat{y}_{ql}\bar{\partial}y^{pq} + \bar{\partial}\hat{y}_{kl}) - \\
&- 2\bar{\partial}y^{kl}(2\hat{y}_{pk}\hat{y}_{ql}\partial y^{pq} + \partial\hat{y}_{kl}) - \frac{i}{2y}C_{\alpha\beta}\Big[\partial\theta^{l\alpha}(i\sqrt{2}\hat{y}_{kl}\bar{\partial}\theta^{k\beta} + \bar{\partial}\hat{\theta}^{\beta}_{l}) - \\
&- (i\sqrt{2}\hat{y}_{kl}\partial\theta^{k\alpha} + \partial\hat{\theta}^{\alpha}_{l})\bar{\partial}\theta^{l\beta}\Big] + \frac{i}{2}yC_{\alpha\beta}(-i\sqrt{2}\hat{y}_{lk}\partial\xi^{l\alpha}\bar{\partial}\xi^{k\beta} + i\sqrt{2}\hat{y}_{lk}\partial\xi^{k\alpha}\bar{\partial}\xi^{l\beta})\Big\}.
\end{aligned} \tag{2.13}$$

The term quadratic in the $\theta^{l\alpha}$ derivatives is multiplied by a three-dimensional antisymmetric matrix, whose rank is two, and the higher order terms in $\hat{\theta}^{\alpha}_{l}$ and $\xi^{l\alpha}$ cannot make the matrix's kernel trivial. Thus the term quadratic in the fermionic gauge fields in the dualized action will be multiplied by a singular matrix and the fermionic gauge fields will be multiplied by a singular matrix in the equations of motion — one cannot T-dualize all the six fermionic coordinates.

Since the obstruction to T-dualizing the fermionic coordinates is at the zeroth order in the spectator fermions, it appears that modifying the $\kappa$-symmetry gauge-fixing of these fermionic degrees of freedom would not change the above conclusion.

## 3 Discussion

We showed that the application of the Buscher T-duality procedure to the coset $\mathrm{OSp}(6|4)/(\mathrm{SO}(2,1)\times\mathrm{U}(3))$ fails when dualizing along the $AdS_4$ flat directions, three of the (real) $\mathbb{C}\mathrm{P}^3$ directions and six fermionic directions. There are several ways to explain this apparent tension between the field theory tree-level evidence and the sigma-model analysis.

The simplest and most obvious explanation is that the dual superconformal symmetry exists only in the weakly-coupled field theory description and breaks down at the strong-coupling regime, which is described by the string theory dual. A second possibility is that in this case the dual superconformal symmetry is not related to the ordinary superconformal symmetry by a T-duality transformation but in a more intricate way.

A third possibility is that the coset formulation does not capture the entire superstring description. The coset is obtained by a partial gauge-fixing of the $\kappa$-symmetry of the full $AdS_4 \times \mathbb{C}\mathrm{P}^3$ sigma-model [16] by setting the fermionic coordinates corresponding to the eight broken supersymmetries to zero. However, as noted in [16], this gauge-fixing is not compatible with all the possible string configurations. Thus, it does not have a representation for certain field theory operators, which might amount to a (possibly inconsistent) truncation of the field theory that does not preserve the dual superconformal symmetry. A way to resolve this issue could be to use a better gauge-fixing of the $\kappa$-symmetry as proposed in [13, 16].

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## A The osp(6|4) superalgebra

The osp(6|4) algebra's commutation relations in the so(1,2)⊕u(3) basis are given by

$$[\lambda_k{}^l, \lambda_m{}^n] = \frac{i}{\sqrt{2}}(\delta_m{}^l \lambda_k{}^n - \delta_k{}^n \lambda_m{}^l), \tag{A.1}$$

$$[\lambda_k{}^l, R_{mn}] = \frac{i}{\sqrt{2}}(\delta_m{}^l R_{kn} - \delta_n{}^l R_{km}), \qquad [\lambda_l{}^k, \hat{R}^{pq}] = -\frac{i}{\sqrt{2}}(\delta_l^p \hat{R}^{kq} - \delta_l^q \hat{R}^{kp}) \tag{A.2}$$

$$[R_{mn}, R_{kl}] = 0, \qquad [R_{mn}, \hat{R}^{kl}] = \frac{i}{\sqrt{2}}(\delta_m{}^k \lambda_n{}^l - \delta_m{}^l \lambda_n{}^k - \delta_n{}^k \lambda_m{}^l + \delta_n{}^l \lambda_m{}^k) \tag{A.3}$$

$$[P_a,P_b]=0,\qquad [K_a,K_b]=0,\qquad [P_a,K_b]=\eta_{ab}D-M_{ab} \tag{A.4}$$

$$[M_{ab},M_{cd}]=\eta_{ac}M_{bd}+\eta_{bd}M_{ac}-\eta_{ad}M_{bc}-\eta_{bc}M_{ad} \tag{A.5}$$

$$[M_{ab},P_c]=\eta_{ac}P_b-\eta_{bc}P_a,\qquad [M_{ab},K_c]=\eta_{ac}K_b-\eta_{bc}K_a \tag{A.6}$$

$$[D,P_a]=P_a,\qquad [D,K_a]=-K_a,\qquad [D,M_{ab}]=0 \tag{A.7}$$

$$[D,Q_{l\alpha}]=\frac{1}{2}Q_{l\alpha},\qquad [D,S_{l\alpha}]=-\frac{1}{2}S_{l\alpha} \tag{A.8}$$

$$[P_a,Q_{l\alpha}]=0,\qquad [K_a,S_{l\alpha}]=0 \tag{A.9}$$

$$[P_a,S_{l\alpha}]=-\frac{i}{\sqrt{2}}(\gamma_a)_\alpha{}^\beta Q_{l\beta},\qquad [K_a,Q_{l\alpha}]=\frac{i}{\sqrt{2}}(\gamma_a)_\alpha{}^\beta S_{l\beta} \tag{A.10}$$

$$[M_{ab},Q_{l\alpha}]=-\frac{i}{2}(\gamma_{ab})_\alpha{}^\beta Q_{l\beta},\qquad [M_{ab},S_{l\alpha}]=-\frac{i}{2}(\gamma_{ab})_\alpha{}^\beta S_{l\beta} \tag{A.11}$$

$$[R_{kl},\hat{Q}^p_\alpha]=\frac{i}{\sqrt{2}}(\delta_l{}^pQ_{k\alpha}-\delta_k{}^pQ_{l\alpha}),\qquad [R_{kl},\hat{S}^p_\alpha]=-\frac{i}{\sqrt{2}}(\delta_l{}^pS_{k\alpha}-\delta_k{}^pS_{l\alpha}) \tag{A.12}$$

$$[\hat{R}^{kl},Q_{p\alpha}]=-\frac{i}{\sqrt{2}}(\delta_p{}^l\hat{Q}^k_\alpha-\delta_p{}^k\hat{Q}^l_\alpha),\qquad [\hat{R}^{kl},S_{p\alpha}]=\frac{i}{\sqrt{2}}(\delta_p{}^l\hat{S}^k_\alpha-\delta_p{}^k\hat{S}^l_\alpha) \tag{A.13}$$

$$[\lambda_k{}^l,Q_{p\alpha}]=\frac{i}{\sqrt{2}}\delta_p{}^lQ_{k\alpha},\qquad [\lambda_k{}^l,S_{p\alpha}]=\frac{i}{\sqrt{2}}\delta^{pl}S_{k\alpha} \tag{A.14}$$

$$[\lambda_k{}^l,\hat{Q}^p_\alpha]=-\frac{i}{\sqrt{2}}\delta_k{}^p\hat{Q}^l_\alpha,\qquad [\lambda_k{}^l,\hat{S}^p_\alpha]=-\frac{i}{\sqrt{2}}\delta_k{}^p\hat{S}^l_\alpha \tag{A.15}$$

$$\{Q_{l\alpha},Q_{k\beta}\}=0,\qquad \{Q_{l\alpha},\hat{Q}^k_\beta\}=-\frac{1}{\sqrt{2}}\delta_l{}^k(\gamma^aC)_{\alpha\beta}P_a \tag{A.16}$$

$$\{S_{l\alpha},S_{k\beta}\}=0,\qquad \{S_{l\alpha},\hat{S}^k_\beta\}=-\frac{1}{\sqrt{2}}\delta_l{}^k(\gamma^aC)_{\alpha\beta}K_a \tag{A.17}$$

$$\{Q_{l\alpha},S_{k\beta}\}=-\frac{1}{\sqrt{2}}C_{\alpha\beta}R_{lk},\qquad \{\hat{Q}^l_\alpha,\hat{S}^k_\beta\}=-\frac{1}{\sqrt{2}}C_{\alpha\beta}\hat{R}^{lk} \tag{A.18}$$

$$\{Q_{l\alpha},\hat{S}^k_\beta\}=-i\frac{1}{2}\delta_l{}^k\left(C_{\alpha\beta}D+i\frac{1}{2}(\gamma^{ab}C)_{\alpha\beta}M_{ab}\right)+\frac{1}{\sqrt{2}}C_{\alpha\beta}\lambda_l{}^k \tag{A.19}$$

$$\{\hat{Q}^l_\alpha,S_{k\beta}\}=i\frac{1}{2}\delta_k{}^l\left(C_{\alpha\beta}D-i\frac{1}{2}(\gamma^{ab}C)_{\alpha\beta}M_{ab}\right)+\frac{1}{\sqrt{2}}C_{\alpha\beta}\lambda_k{}^l \tag{A.20}$$

The indices take the values $k,l=1,\dots,3$, the $\mathbf{3}$ u(3), $a,b=0,1,2$ are the $\mathbf{3}$ of so(1,2) and $\alpha,\beta,\dots=1,2$ are the so(2,1) spinors, and $\eta=\mathrm{diag}(-,+,+)$. The generators satisfy the following relations under complex conjugation $R^*_{kl}=\hat{R}^{kl}$, $\lambda_k{}^l=\lambda_l^{*k}$, $\hat{Q}^l_\alpha=(Q_{l\alpha})^*$ and $\hat{S}^l_\alpha=(S_{l\alpha})^*$. The $(\gamma_a)_\alpha{}^\beta$ are the Dirac matrices of so(1,2), and $\gamma_{ab}=\frac{i}{2}[\gamma_a,\gamma_b]$. We raise and lower spinor indices using $C_{\alpha\beta}=\epsilon_{\alpha\beta}$, $\psi_\alpha=\psi^\beta\epsilon_{\beta\alpha}$, $\psi^\alpha=\epsilon^{\alpha\beta}\psi_\beta$, where $\epsilon_{12}=-\epsilon_{21}=\epsilon^{12}=-\epsilon^{21}=1$.



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The bilinear forms are given by

$$\begin{aligned} \mathrm{Str}(R_{kl}, \hat{R}^{pq}) &= \delta_k{}^q \delta_l{}^p - \delta_k{}^p \delta_l{}^q, \\ \mathrm{Str}(\lambda_k{}^l, \lambda_p{}^q) &= -\delta_k{}^q \delta_l{}^p, \\ \mathrm{Str}(Q_{l\alpha}, \hat{S}^k_\beta) &= i\delta_l{}^k C_{\alpha\beta}, \\ \mathrm{Str}(S_{l\alpha}, \hat{Q}^k_\beta) &= -i\delta_k{}^l C_{\alpha\beta}, \\ \mathrm{Str}(P_a, K_b) &= -\eta_{ab}, \\ \mathrm{Str}(D, D) &= -1, \\ \mathrm{Str}(M_{ab}, M_{cd}) &= \eta_{ac}\eta_{bd} - \eta_{ad}\eta_{bc}. \end{aligned} \tag{A.21}$$

The $\mathbb{Z}_4$ subspaces with the invariant locus of $\mathrm{U}(3) \times \mathrm{SO}(3,1)$ which gives the semi-symmetric space $AdS_4 \times \mathbb{C}\mathrm{P}^3$ are

$$\begin{aligned} \mathcal{H}_0 &= \{P_a - K_a, M_{ab}, \lambda_k{}^l\}, \\ \mathcal{H}_1 &= \{Q_{l\alpha} - S_{l\alpha}, \hat{Q}^l_\alpha - \hat{S}^l_\alpha\}, \\ \mathcal{H}_2 &= \{P_a + K_a, D, R_{kl}, \hat{R}^{kl}\}, \\ \mathcal{H}_3 &= \{Q_{l\alpha} + S_{l\alpha}, \hat{Q}^l_\alpha + \hat{S}^l_\alpha\}. \end{aligned} \tag{A.22}$$

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