The cosmological constant: a lesson from Bose–Einstein condensates

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The cosmological constant is one of the most pressing problems in modern physics. In this Letter, we address the issue of its nature and computation using an analogue gravity standpoint as a toy model for an emergent gravity scenario. Even if it is well known that phonons in some condense matter systems propagate like a quantum field on a curved spacetime, only recently it has been shown that the dynamics of the analogue metric in a Bose–Einstein condensate can be described by a Poisson-like equation with a vacuum source term reminiscent of a cosmological constant. Here we directly compute this term and confront it with the other energy scales of the system. On the gravity side of the analogy, this model suggests that in emergent gravity scenarios it is natural for the cosmological constant to be much smaller than its naïf value computed as the zero-point energy of the emergent effective field theory. The striking outcome of our investigation is that the value of this constant cannot be easily predicted by just looking at the ground state energy of the microscopic system from which spacetime and its dynamics should emerge. A proper computation would require the knowledge of both the full microscopic quantum theory and a detailed understanding about how Einstein equations emerge from such a fundamental theory. In this light, the cosmological constant appears even more a decisive test bench for any quantum/emergent gravity scenario.

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The cosmological constant [1] has been one of the most mysterious and fascinating objects for both cosmologist and theoretical physicists since its introduction eighty years ago [2]. Once called by Einstein his greatest blunder, it seems nowadays the driving force behind the current accelerated expansion of the universe. The explanation of its origin is considered one of the most fundamental issues for our comprehension of general relativity (GR) and quantum field theory.

Since this constant appears in Einstein equations as a source term present even in the absence of matter and with the symmetries of the vacuum ($T^{\Lambda}_{\mu\nu} \propto g_{\mu\nu}$), it is usually interpreted as a "vacuum energy". Unfortunately, this has originated the so-called "worst prediction" of physics. In fact, the estimated value, which is naïvely obtained by integrating the zero-point energies of modes of quantum fields below Planck energy, is about 120 orders of magnitude larger than the measured value. Despite the large number of attempts (most notably supersymmetry [3], which, however, must be broken at low energy) this problem is still open. We can summarize the situation by saying that, given the absence of custodial symmetries protecting the cosmological term from large renormalization effects, the only option we have to explain observations is fine tuning [5, 6].

This huge discrepancy is plausibly due to the use of effective field theory (EFT) calculations for a quantity which can be computed only within a full quantum theory of gravity. Unfortunately, to date, we do not have any conclusive theory at our disposal. However, the possibility of a failure of our EFT-based intuition is supported by what can be learned from analogue models of gravity [7], given that, in these models, the way in which the structure of the spacetime emerges from the microscopic theory is fully under control. In [8, 9] it was shown that a naïf computation of the ground state energy using the effective field theory (the analogue that one would do to compute the cosmological constant), would produce a wrong result. The unique way to compute the correct value seems to use the full microscopic theory.

Given the deep difference in the structure of the equations of fluid dynamics and those of GR (and other gravitational theories) it is not possible to have an accurate analogy at the dynamical level: indees, this is forbidden by the absence of diffeomorphism invariance and of local Lorentz invariance. However, in [10] it has been shown for the first time that the evolution of part of the acoustic metric in a Bose–Einstein condensate (BEC) is described by a Poisson equation for a nonrelativistic gravitational field, thus realizing a (partial) dynamical analogy with Newtonian gravity. Noticeably, this equation is endowed with a source term which is there even in the absence of real phonons and can be naturally identified as a cosmological constant.

In this Letter we will consider such analogue model for gravity and directly show that the cosmological constant term cannot be computed through the standard effective field theory approach, confirming the conjecture of [8]. However, we find that also the total ground state energy of the condensate does not give the correct result: indeed, the cosmological constant is comparable with that fraction of the ground state energy corresponding to the quantum depletion of the condensate, *i.e.* to the fraction of atoms inevitably occupying excited states of the single particle Hamiltonian. In conclusion, the origin and value of such term teach us some interesting lessons about the cosmological constant in emergent gravity scenarios.

Settings — The model used in [10] is a modified BEC system including a soft breaking of the $U(1)$ symmetry associated with the conservation of particle number. This unusual choice is a simple trick to give mass to quasiparticles that are otherwise massless by Goldstone's theorem. In second quantization, such a system is described by a canonical field $\hat{\Psi}^\dagger$, satisfying $[\hat{\Psi}(t,\mathbf{x}),\hat{\Psi}^\dagger(t,\mathbf{x}')] = \delta^3(\mathbf{x}-\mathbf{x}')$, whose dynamics is generated by the Grand Canonical Hamiltonian $\hat{\mathcal{H}} = \hat{H} - \mu\hat{N}$, where

$$\hat{H} = \int d^3x \left[\frac{\hbar^2}{2m}\nabla^2\hat{\Psi}^\dagger\,\nabla^2\hat{\Psi} + V\hat{\Psi}^\dagger\hat{\Psi} + \frac{g}{2}\hat{\Psi}^\dagger\hat{\Psi}^\dagger\hat{\Psi}\hat{\Psi} - \frac{\lambda}{2}\left(\hat{\Psi}\hat{\Psi} + \hat{\Psi}^\dagger\hat{\Psi}^\dagger\right)\right], \quad (1)$$

and $\hat{N}$ is the standard number operator for $\hat{\Psi}$. For more details on this model and on possible physical realizations, see [10]. See also [16] for a generalization to condensates with many components.

We describe the formation of a BEC at low temperature through a complex function Ψ_0 for the condensate and an operator $\hat{\phi}$ for the perturbations on top of it [13]:

$$\hat{\Psi} = \Psi_0(\mathbb{I} + \hat{\phi}). \quad (2)$$

The canonical commutation relation for $\hat{\Psi}^\dagger$ implies

$$\left[\hat{\phi}(t,x),\hat{\phi}^\dagger(t,x')\right] = \frac{1}{\rho_0(x)}\delta(x-x'). \quad (3)$$

We adopt the notation of [14], where a rigorous quantization and mode analysis of the field $\hat{\phi}$ is presented for a standard BEC. Those results are here summarized and generalized to the $U(1)$-breaking case of [10].

For a stationary condensate, $\partial_t\Psi_0 = 0$ and Eqs. (1) and (2) lead to a modified Gross–Pitaëvski equation

$$\left[-\frac{\hbar^2}{2m}\partial_x{}^2 + V - \mu + g\rho_0 - \lambda\frac{\Psi_0{}^*}{\Psi_0}\right]\Psi_0 = 0. \quad (4)$$

For the aim of this Letter, it is enough to consider only homogeneous backgrounds. Thus, one can assume that $V = 0$ and the condensate is at rest, such that Ψ_0 has a constant phase. For stability reasons, Ψ_0 must be real ($\Psi_0{}^* = \Psi_0 = \sqrt{\rho_0}$), and Eq. (4) simplifies to $\mu = g\rho_0 - \lambda$.

The equation for the quasiparticles is solved via Bogoliubov transformation involving the Fourier expansion

$$\hat{\phi} = \int \frac{d^3k}{\sqrt{\rho_0(2\pi)^3}}\left[u_{\mathbf{k}}e^{-i\omega t + i\mathbf{k}\cdot x}\hat{a}_k + v_{\mathbf{k}}^* e^{+i\omega t - i\mathbf{k}\cdot x}\hat{a}_k^\dagger\right], \quad (5)$$

where $\hat{a}_k$ and $\hat{a}_k^\dagger$ are quasiparticles' operators and the factor $\sqrt{\rho_0(2\pi)^3}$ has been inserted such that the Bogoliubov coefficients $u_{\mathbf{k}}$ and $v_{\mathbf{k}}$ obey the standard normalization $|u_{\mathbf{k}}|^2 - |v_{\mathbf{k}}|^2 = 1$.

The dispersion relation is

$$\hbar^2\omega^2 = 4\lambda g\rho_0 + \frac{g\rho_0 + \lambda}{m}\hbar^2k^2 + \frac{\hbar^4k^4}{4m^2}, \quad (6)$$

describing massive phonons with ultraviolet corrections, mass $\mathcal{M}$, and speed of sound c_s [10]

$$\mathcal{M} = \frac{2\sqrt{\lambda g\rho_0}}{g\rho_0 + \lambda}m, \qquad c_s^2 = \frac{g\rho_0 + \lambda}{m}. \quad (7)$$

Finally, standard manipulations give

$$u_{\mathbf{k}}{}^2 = \frac{1}{1 - D_{\mathbf{k}}^2}, \qquad v_{\mathbf{k}}{}^2 = \frac{D_{\mathbf{k}}^2}{1 - D_{\mathbf{k}}^2}, \quad (8)$$

$$D_{\mathbf{k}} \equiv \frac{\hbar\omega - \left(\hbar^2\mathbf{k}^2/2m + g\rho_0 + \lambda\right)}{g\rho_0 - \lambda}, \quad (9)$$

where both $u_{\mathbf{k}}$ and $v_{\mathbf{k}}$ are chosen to be real.

Vacuum expectation values — We can now compute the vacuum expectation value of $\hat{\mathcal{H}}$ in the ground state $|\Omega\rangle$, the Fock vacuum of the quasiparticles ($\hat{a}_k|\Omega\rangle = 0,\ \forall k$). To this aim, it is convenient to expand $\hat{\mathcal{H}}$ in powers of $\hat{\phi}$: $\hat{\mathcal{H}} \approx \mathcal{H}_0 + \hat{\mathcal{H}}_1 + \hat{\mathcal{H}}_2$, where $\mathcal{H}_0$, $\hat{\mathcal{H}}_1$, and $\hat{\mathcal{H}}_2$ contain respectively no power of $\hat{\phi}$, only first powers, and only second powers, and higher order terms associated with quasiparticles' self-interactions are neglected. The energy density h_0 of the condensate (density of $\mathcal{H}_0$) and the density h_2 of the expectation value of $\hat{\mathcal{H}}_2$ are

$$h_0 = -\frac{g\rho_0{}^2}{2}, \qquad h_2 = -\int\frac{d^3k}{(2\pi)^3}\hbar\omega|v_{\mathbf{k}}|^2, \quad (10)$$

while the expectation value of $\hat{\mathcal{H}}_1$ vanishes because it contains only odd powers of $\hat{a}_k$ and $\hat{a}_k^\dagger$. The integral in Eq. (10) is computed by using Eqs. (8) and (9). Applying standard regularization techniques (see also [15])

$$h_2 = \frac{64}{15\sqrt{\pi}}g\rho_0{}^2\sqrt{\rho_0 a^3}\ F_h\left(\frac{\lambda}{g\rho_0}\right), \quad (11)$$

where $a = 4\pi gm/\hbar^2$ is the scattering length, F_h is plotted in Fig. 1 (dashed line) and $F_h(0)$=1.

The total grand-canonical energy density is therefore

$$h = h_0 + h_2 = \frac{g\rho_0{}^2}{2}\left[-1 + \frac{128}{15\sqrt{\pi}}F_h\left(\frac{\lambda}{g\rho_0}\right)\right] \quad (12)$$

and it coincides with the well known Lee–Huang–Yang formula [11] when $\lambda = 0$.

The number density operator $\hat{N}$ is analogously expanded in powers of $\hat{\phi}$: $\hat{N} = N_0 + \hat{N}_1 + \hat{N}_2$. The density of N_0 is simply $\rho_0 = |\Psi_0|^2$, $\langle\hat{N}_1\rangle_\Omega = 0$, and the density $\rho_2 \equiv \langle\hat{N}_2\rangle_\Omega$ is

$$\rho_2 = \rho_0\langle\hat{\phi}^\dagger\hat{\phi}\rangle_\Omega = \int\frac{d^3k}{(2\pi)^3}|v_{\mathbf{k}}|^2 = \frac{8\rho_0}{3\sqrt{\pi}}\sqrt{\rho_0 a^3}\ F_\rho\left(\frac{\lambda}{g\rho_0}\right), \quad (13)$$

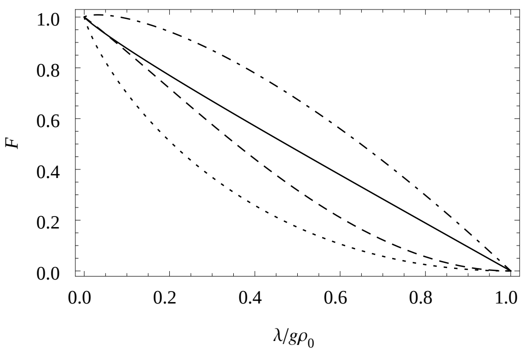

FIG. 1: F_h [dashed line, Eq. (11)], F_ρ [dotted line, Eq. (13)], $F_{\phi\phi}$ [dotdashed line, Eq. (22)], and F_Λ [solid line, Eq. (23)].

where F_ρ satisfies $F_\rho(0) = 1$ (see Fig. 1, dotted line). This is the number density of non-condensed atoms (*depletion*). Note that $\rho_0 a^3$ is the so called dilution factor which has to be much smaller than one for the Hamiltonian (1) to hold.

Furthermore, when $\lambda = 0$, inverting the expression for total particle density, $\rho = \rho_0 + \rho_2$, one obtains, up to the first order in $\sqrt{\rho a^3}$

$$\rho_0 = \rho \left[1 - \frac{8}{3\sqrt{\pi}}\sqrt{\rho a^3}\right], \tag{14}$$

which is the density of condensed atoms in terms of the total density ρ and the scattering length a [11]. In this case, $\mu = g\rho_0$, such that the energy density ϵ (density of $\langle \hat{H} \rangle_\Omega = \langle \mathcal{H} + \mu \hat{N} \rangle_\Omega$) is

$$\epsilon = h + \mu\rho = \frac{g\rho^2}{2}\left[1 + \frac{128}{15\sqrt{\pi}}\sqrt{\rho a^3}\right]. \tag{15}$$

This is the well known Lee–Huang–Yang [11] formula for the ground state energy in a condensate at zero temperature. In general, when the $U(1)$ breaking term is small, this term is expected to be the dominant contribution to the ground state energy of the condensate.

Analogue cosmological constant — When the homogeneous condensate background is perturbed by small inhomogeneities, the Hamiltonian for the quasi-particles can be written as (see [10])

$$\hat{H}_{quasip.} \approx \mathcal{M}c_s^2 - \frac{\hbar^2\nabla^2}{2\mathcal{M}} + \mathcal{M}\Phi_{\rm g}. \tag{16}$$

$\hat{H}_{quasip.}$ is the non-relativistic Hamiltonian for particles of mass $\mathcal{M}$ [see Eq. (7)] in a gravitational potential

$$\Phi_{\rm g}(\mathbf{x}) = \frac{(g\rho_0 + 3\lambda)(g\rho_0 + \lambda)}{2\lambda m} u(\mathbf{x}) \tag{17}$$

and $u(\mathbf{x}) = [(\rho_0(\mathbf{x})/\rho_\infty) - 1]/2$, where ρ_∞ is the asymptotic density of the condensate. Moreover, the dynamics of the potential $\Phi_{\rm g}$ is described by a Poisson-like equation

$$\left[\nabla^2 - \frac{1}{L^2}\right]\Phi_{\rm g} = 4\pi G_N \rho_{\rm p} + C_\Lambda, \tag{18}$$

which is the equation for a non-relativistic short-range field with length scale L and gravitational constant G_N:

$$L = \frac{a}{\sqrt{16\pi\rho_0 a^3}}, \quad G_{\rm N} = \frac{g(g\rho_0 + 3\lambda)(g\rho_0 + \lambda)^2}{4\pi\hbar^2 m\lambda^{3/2}(g\rho_0)^{1/2}}. \tag{19}$$

Despite the obvious difference between $\Phi_{\rm g}$ and the usual Newtonian gravitational potential, we insist in calling it the *Newtonian potential* because it enters the acoustic metric exactly as the Newtonian potential enters the metric tensor in the Newtonian limit of GR.

The source term in Eq. (18) contains both the contribution of real phonons (playing the role of matter)

$$\rho_{\rm p} = \mathcal{M}\rho_0 \left[\left(\langle \hat{\phi}^\dagger \hat{\phi} \rangle_\zeta - \langle \hat{\phi}^\dagger \hat{\phi} \rangle_\Omega\right) + \frac{1}{2}{\rm Re}\left(\langle \hat{\phi}\hat{\phi} \rangle_\zeta - \langle \hat{\phi}\hat{\phi} \rangle_\Omega\right)\right], \tag{20}$$

where $|\zeta\rangle$ is some state of real phonons, as well as a cosmological constant like term (present even in the absence of phonons/matter)

$$C_\Lambda = \frac{2g\rho_0(g\rho_0 + 3\lambda)(g\rho_0 + \lambda)}{\hbar^2\lambda}{\rm Re}\left[\langle \hat{\phi}^\dagger \hat{\phi} \rangle_\Omega + \frac{1}{2}\langle \hat{\phi}\hat{\phi} \rangle_\Omega\right]. \tag{21}$$

Note that the source term in the correct weak field approximation of Einstein equations is $4\pi G_N(\rho + 3p/c^2)$. For standard nonrelativistic matter, p/c^2 is usually negligible with respect to ρ. However, it cannot be neglected for the cosmological constant, since $p_\Lambda/c^2 = -\rho_\Lambda$. As a consequence $C_\Lambda = -2c_s^2\Lambda$, where Λ would be the GR cosmological constant. From Eq. (13) and evaluating

$$\langle \hat{\phi}\hat{\phi} \rangle_\Omega = \int \frac{d^3k}{\rho_0(2\pi)^3} u_{\mathbf{k}} v_{\mathbf{k}} = \frac{8}{\sqrt{\pi}}\sqrt{\rho_0 a^3}\, F_{\phi\phi}\left(\frac{\lambda}{g\rho_0}\right), \tag{22}$$

where $F_{\phi\phi}(0) = 1$ (see Fig. 1, dotdashed line), we obtain

$$\Lambda = -\frac{20m\, g\rho_0\,(g\rho_0 + 3\lambda)}{3\sqrt{\pi}\hbar^2\lambda}\sqrt{\rho_0 a^3}\, F_\Lambda\left(\frac{\lambda}{g\rho_0}\right), \tag{23}$$

where $F_\Lambda = (2F_\rho + 3F_{\phi\phi})/5$ (see Fig. 1, solid line).

Let us now compare the value of Λ either with the ground-state grand-canonical energy density h [Eq. (12)], which in [8] was suggested as the correct vacuum energy corresponding to the cosmological constant, or to the ground-state energy density ϵ of Eq. (15). Evidently, Λ does not correspond to either of them: even when taking into account the correct behavior at small scales, the vacuum energy computed with the phonon effective field theory does not lead to the correct value of the cosmological constant appearing in Eq. (18). Noticeably, since Λ is proportional to $\sqrt{\rho_0 a^3}$, it can even be arbitrarily smaller both than h and than ϵ, if the condensate is very dilute. Furthermore, Λ is proportional only to the subdominant second order correction of h or ϵ, which is strictly related to the depletion [see Eq. (13)].

Fundamental scales — Several scales show up in the emergent system, in addition to the naïf Planck scale computed by combining the emergent constants $G_{\rm N}$, c_s and $\hbar$:

$$L_{\rm P} = \sqrt{\frac{\hbar c_s^5}{G_{\rm N}}} \propto \left(\frac{\lambda}{g\rho_0}\right)^{-3/4} (\rho_0 a^3)^{-1/4} a. \tag{24}$$

For instance, the Lorentz-violation scale (i.e., the healing lenght of the condensate) $L_{\rm LV} = \xi \propto (\rho_0 a^3)^{-1/2} a$ differs from $L_{\rm P}$, suggesting that the breaking of the Lorentz symmetry might be expected at scale much longer than the Planck length (energy much smaller than the Plack energy), since the ratio $L_{\rm LV}/L_{\rm P} \propto (\rho_0 a^3)^{-1/4}$ increases with the diluteness of the condensate.

Note that $L_{\rm LV}$ scales with $\rho_0 a^3$ exactly as the range of the gravitational force [see Eq. (19)], signaling that this model is too simple to correctly grasp all the desired features. However, in more complicated systems [16], this pathology can be cured, in the presence of suitable symmetries, leading to long range potentials.

It is instructive to compare the energy density corresponding to Λ to the Planck energy density:

$$\mathcal{E}_\Lambda = \frac{\Lambda c_s^4}{4\pi G_{\rm N}}, \quad \mathcal{E}_{\rm P} = \frac{c_s^7}{\hbar G_{\rm N}^2}, \quad \frac{\mathcal{E}_\Lambda}{\mathcal{E}_{\rm P}} \propto \rho_0 a^3 \left(\frac{\lambda}{g\rho_0}\right)^{-5/2}. \tag{25}$$

The energy density associated with the analogue cosmological constant is much smaller than the values computed from zero-point-energy calculations with a cut off at the Planck scale. Indeed, the ratio between these two quantities is controlled by the diluteness parameter $\rho_0 a^3$.

Final remarks — Taken at face value, this relatively simple model displays too many crucial differences with any realistic theory of gravity to provide conclusive evidences. However, it displays an alternative path to the cosmological constant, from the perspective of a microscopic model. The analogue cosmological constant that we have discussed *cannot* be computed as the total zero-point energy of the condensed matter system, even when taking into account the natural cut-off coming from the knowledge of the microphysics [8]. In fact the value of Λ is related only to the (subleading) part of the zero-point energy proportional to the quantum depletion of the condensate.

The implications for gravity are twofold. First, there could be no *a priori* reason why the cosmological costant should be computed as the zero-point energy of the system. More properly, its computation must inevitably pass through the derivation of Einstein equations emerging from the underlying microscopic system. Second, the energy scale of Λ can be several orders of magnitude smaller than all the other energy scales for the presence of a very small number, nonperturbative in origin, which cannot be computed within the framework of an effective field theory dealing only with the emergent degrees of freedom (*i.e.* semiclassical gravity).

The model discussed in this Letter shows all this explicitly. Furthermore, it strongly supports a picture where gravity is a collective phenomenon in a pregeometric theory. In fact, the cosmological constant puzzle is elegantly solved in those scenarios. From an emergent gravity approach, the low energy effective action (and its renormalization group flow) is obviously computed within a framework that has nothing to do with quantum field theories in curved spacetime. Indeed, if we interpreted the cosmological constant as a coupling constant controlling some self-interaction of the gravitational field, rather than as a vacuum energy, it would straightforwardly follow that the explanation of its value (and of its properties under renormalization) would naturally sit outside the domain of semiclassical gravity.

For instance, in a group field theory scenario (a generalization to higher dimensions of matrix models for two dimensional quantum gravity [19]), it is transparent that the origin of the gravitational coupling constants has nothing to do with ideas like “vacuum energy” or statements like “energy gravitates”, because energy *itself* is an emergent concept. Rather, the value of Λ is determined by the microphysics, and, most importantly, by the procedure to approach the continuum semiclassical limit. In this respect, it is conceivable that the very notion of cosmological constant as a form of energy intrinsic to the vacuum is ultimately misleading. To date, little is known about the macroscopic regime of models like group field theories, even though some preliminary steps have been recently done [20]. Nonetheless, analogue models elucidate in simple ways what is expected to happen and can suggest how to further develop investigations in quantum gravity models. In this respect, the reasoning of this Letter sheds a totally different light on the cosmological constant problem, turning it from a failure of effective field theory to a question about the emergence of the spacetime.

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