Particles and Fields in Radio Galaxies
ASP Conference Series, Vol. , 2001
Robert A. Laing and Katherine M. Blundell

Acceleration by relativistic shock fronts

J. G. Kirk
Max-Planck-Institut für Kernphysik, Postfach 10 39 80, 69027 Heidelberg, Germany

Abstract. Both a semi-analytic treatment and Monte-Carlo simulations of the problem of test particle acceleration at an ultra-relativistic shock predict a power-law spectrum of index $\mathrm{d}\ln f/\mathrm{d}\ln p \approx -4.2$, where f is the phase space density and p the particle momentum. A brief review is given of this result, together with a discussion of its robustness and relevance to observations.

1. Introduction

More than a decade after the original papers, there has recently been a renewal of interest in the problem of particle acceleration at relativistic shock fronts. This is mainly attributable to the role that ultra-relativistic shock fronts play in the theory of gamma-ray bursts, but is also in part due to the relatively high Lorentz factors discussed in connection with rapidly variable gamma-ray blazars and with radio sources which show intra-day variability. Consequently, a change of emphasis is apparent. Whereas early work (e.g., Kirk & Schneider 1987; Heavens and Drury 1988) was concerned with shocks of Lorentz factor $\Gamma \lesssim 5$, recent studies have looked at $\Gamma \sim$ hundreds, or even the limit $\Gamma \to \infty$.

Apart from particle-in-cell simulations of 1.5 dimensional relativistic plasmas (Hoshino et al 1992; Smolsky & Usov 2000), which cover a limited range of particle energies, the theoretical approach to the problem has almost invariably been to use a highly simplified model of particle transport, and assume the accelerated particles can be treated as test particles interacting with a background fluid flow (for a review see Kirk & Duffy 1999). Despite the obvious limitations involved, this method predicts the power-law index expected of particles accelerated to Lorentz factors well above that of the shock front, and so enables, at least in principle, a comparison with observation. This paper presents a brief summary of the results obtained by both semi-analytic methods of kinetic theory and by Monte-Carlo simulation. More detailed discussions can be found in Kirk et al (2000) and Achterberg et al (2001)

2. The test-particle problem

Consider an ideal magneto-hydrodynamic flow in which a shock front is embedded. The magnetic field and flow velocity both up and downstream of the shock is constant in space and time. The orbit of a test particle which interacts only with the magnetic field is completely determined and depends on whether the shock front is super- or subluminal (see Kirk, Melrose & Priest 1994). For a subluminal shock, there exists a frame of reference in which the flow velocity is parallel to the magnetic field (a 'de Hofmann/Teller' frame), and, in the absence of scattering, the test particle undergoes no change in energy. On the other hand, a superluminal shock can be viewed with the magnetic field perpendicular to the shock normal. In this case, the average particle energy changes where the orbit intersects the shock front, which usually leads to an increase in energy. However, in the absence of stochastic effects, the energy increase is finite (Begelman & Kirk 1990) and the spectrum of emerging particles depends on the input spectrum. This kind of acceleration is commonly referred to as 'shock-drift' acceleration.

In contrast, the first-order Fermi process (one example of which is diffusive shock acceleration) requires stochasticity of the particle orbits. Either fluctuations of the magnetic field embedded in the plasma, or self-generated turbulence (i.e., generated by the accelerating particles) can provide this. Diffusive shock acceleration makes the assumption that the stochasticity enforces spatial diffusion on the test particles. At relativistic shocks, this assumption is inconsistent, because it requires the particle distribution to be almost isotropic when seen from the fluid rest frames in both the upstream and downstream regions. Because the relative velocity of these frames is almost equal to the speed of light, the requirement cannot be fulfilled. Instead, it is usually assumed that the particles repeatedly undergo small random deflections, leading to diffusion of the direction of motion.

In the conventional picture, pitch-angle diffusion is accompanied by a relatively slow diffusion of the particle's guiding centre across the magnetic field lines (see, for example, Michalek & Ostrowski 1998). For subluminal shocks, neglecting this process does not change the basic physical picture. However, in the case of superluminal shocks, it does, since no stochastic acceleration at all is possible without cross-field transport. Highly relativistic shocks are almost certain to be superluminal, since boosting the upstream magnetic field into the shock frame enhances the component perpendicular to the shock normal, whilst leaving the parallel component unchanged.

Inclusion of cross-field transport is not straightforward, since standard treatments rely on the concept of the trajectory's guiding centre. This is not well-defined in the case of a particle at a highly relativistic shock, since there a trajectory undergoes only a small number of shock crossings, separated in time by less than a gyro period (Begelman & Kirk 1990). Instead of selecting those fluctuations with a wavelength in resonance with the gyro radius, such particles respond to smaller wavelength fluctuations.

Two ways of circumventing this problem in Monte-Carlo simulations have been employed. Bednarz & Ostrowski (1998) use a scheme in which the effects of cross-field transport and pitch-angle diffusion are simulated by a sequence of small, random deflections of the particle's orbit in the regular magnetic field, whereas Gallant, Achterberg & Kirk (1999) assume that short-wavelength fluctuations overwhelm the effect of the regular magnetic field, resulting in isotropic diffusion of the direction of the particle's velocity. The results obtained by these different methods do not differ substantially. However, the latter approach is amenable to an semi-analytic treatment, analogous to that of Kirk & Schneider (1987).

3. Semi-analytic solution

When short-wavelength fluctuations dominate over the regular magnetic field, the equation to be solved is formally identical to that for pitch-angle diffusion at a parallel shock front and reads:

$$\Gamma(u+\mu)\frac{\partial f(p,\mu,z)}{\partial z} = \frac{\partial}{\partial\mu}\left[D_{\mu\mu}\frac{\partial f(p,\mu,z)}{\partial\mu}\right] \quad (1)$$

where u (>0) is the speed of the plasma in units of the speed of light, measured in the shock frame (in which the distribution is assumed stationary), $\Gamma = (1-u^2)^{-1/2}$, $D_{\mu\mu}$ is the diffusion coefficient in angle and $f(p,\mu,z)$ is the (Lorentz invariant) phase-space density, assumed to be a function of the momentum p, the cosine μ of the angle between the particle velocity and the shock normal and a single Cartesian coordinate z. Both the momentum and angle variables are measured in the local rest frame of the plasma, whereas position is measured relative to the shock front in the shock frame.

This equation, with the appropriate values of u and $D_{\mu\mu}$ is valid in both the upstream ($z<0$) and downstream ($z>0$) media; the solutions are required to match at the shock front according to Liouville's theorem. The method used by Kirk & Schneider (1987) and Heavens & Drury (1988) consists of observing that the p dependence of the desired solution is of power-law form: $f(p,\mu,z) = p^{-s}g(\mu,z)$. Then, the z and μ dependencies of g are separated, and the angular distribution is expanded in the eigenfunctions Q_i of the equation:

$$\Lambda_i(u+\mu)Q_i = \frac{\partial}{\partial\mu}\left[D_{\mu\mu}\frac{\partial Q_i}{\partial\mu}\right] \quad (2)$$

where Λ_i are the eigenvalues. This equation has two families of discrete eigenfunctions; one of them with positive Λ_i, the other with negative Λ_i. The former give solutions which diverge as $z\to\infty$, the latter solutions which diverge as $z\to-\infty$. If one wishes to represent the angular distribution in the downstream region, the $\Lambda_i<0$ family is appropriate. This was the approach of Kirk & Schneider (1987) and Heavens & Drury (1988). On the other hand, if one tries to approximate the distribution function upstream, the $\Lambda_i>0$ family is required. Either method may be used to represent the distribution at the shock front. The power-law index s of accelerated particles follows by eliminating the strongest divergent terms in the opposite half-space.

It turns out that, for relativistic shocks, the expansion of the upstream angular distribution converges much faster than the expansion of the downstream distribution – a single term being in most cases sufficient. Figure 1 shows results obtained using this method for relativistic shocks in fluids with different equations of state: that of a purely relativistic gas, and that of a gas with an adiabatic index of 4/3 (for details see Kirk et al 2000). In the limit of an ultra-relativistic shock, s converges to the value 4.23, corresponding to a synchrotron index for uncooled particles of 0.62.

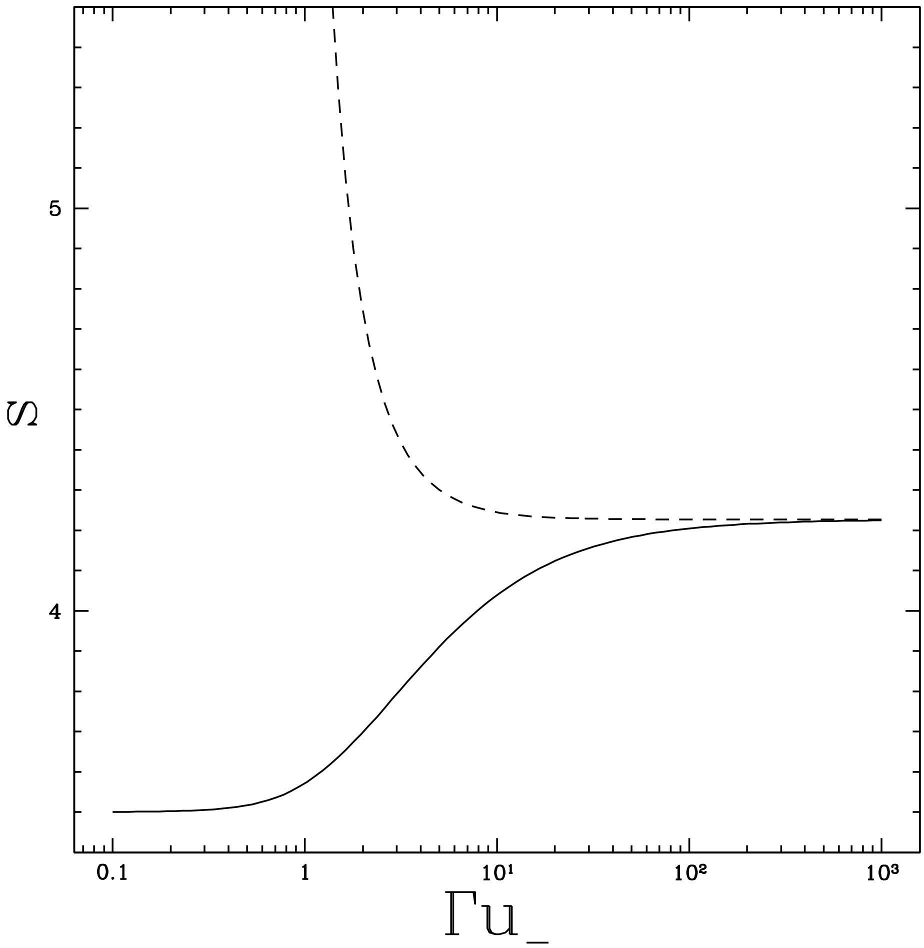

Figure 1. The power-law index s as a function of the spatial component of the upstream shock speed $\Gamma_{-}u_{-}$ for a strong shock with a fixed adiabatic index of 4/3 (solid line) and for a shock in a relativistic gas. The apparent asymptotic value of $s = 4.23$ agrees with explicit computations performed in the limit $u_{-} \rightarrow 1$.

4. Monte-Carlo results

Four different groups have presented simulations of acceleration at ultra-relativistic shocks. Bednarz & Ostrowski (1998) combine a pitch-angle scattering operator with motion in a regular magnetic field and give results for shocks of Lorentz factor up to about 250. Baring (1999) uses both pitch-angle diffusion and large angle-scattering at a parallel shock, presenting results for Lorentz factors up to 80. Gallant et al (2000) assume short scale fluctuations dominate and present simulation results which are valid in the ultra-relativistic limit and Achterberg et al (2001) adopt the short scale fluctuation model downstream and either motion in a regular magnetic field, or short scale fluctuations upstream. Except for the method of Gallant et al (2000), all of these simulations require more computing time for larger Lorentz factors. This is because particles upstream occupy a very small cone of angles close to the shock normal. In order to simulate diffusion in angle by a large number of small deflections within such a small cone, a very short step length must be imposed.

The striking aspect of these simulations is that the asymptotic value of the power-law index is in almost cases close to 4.2. The only exception is that of large-angle scattering, for which there is no convincing physical justification. Differences in the details of the results are, however, apparent. In particular, it appears difficult to obtain an accurate simulation of the angular dependence of the distribution function at the shock front, especially close to the point where particles graze the front, $\mu = -u$. In part, this is due to the poor statistics associated with such particles. However, there is a deeper reason connected with the discretisation of the angular diffusion process (Achterberg et al 2001). Fortunately, grazing incidence particles have very little influence on the acceleration process, since they suffer only a small energy boost on crossing the shock front. This means that one can be much more confident in the results for the particle spectrum than in those for the angular distribution near $\mu = -u$.

5. Discussion

Both semi-analytic and Monte-Carlo results agree that the power-law index produced by ultra-relativistic shocks in the asymptotic (large Lorentz factor) limit is close to 4.2. How robust is this result?

Clearly, it is closely connected with the fact that the asymptotic value of the compression ratio at an ultra-relativistic shock is 3. If the downstream plasma contains a dynamically important magnetic field, the compression is weakened, and the predicted index is softer (Kirk et al 2000). The result also depends to some degree on the type of scattering assumed. Strongly anisotropic diffusion was checked by Kirk et al (2000), and found to have a marked effect for $\Gamma \sim$ few. However, the change found in the ultra-relativistic limit was rather small. A minor difference was also noted by Achterberg et al (2001) between the case in which the upstream transport includes scattering, and that in which unperturbed motion in a regular field is assumed. From this, one could conclude that the nature of the upstream transport is unimportant, provided large deflections are excluded. However, Ostrowski & Bednarz (2000) note that the result is insensitive to the presence or absence of *downstream* scattering. The nature of the downstream scattering would therefore also appear to be unimportant. Of course, if all scattering is removed, the situation reverts to that of shock-drift acceleration (Begelman & Kirk 1990), which does not produce a characteristic power-law spectrum. Stochasticity is required, but it seems that almost any kind of small-angle deflection suffices to give $s \approx 4.2$.

Robustness, however, requires not only insensitivity to parameters, but also plausibility of the underlying assumptions. Perhaps the most important of these is the test-particle picture. In contrast to the situation at strong non-relativistic shocks, the predicted spectrum for ultra-relativistic shocks is softer than $s = 4$. This implies that it is always possible to choose a threshold particle energy, above which the accelerated particles have a negligible contribution to the overall dynamics. If these particles also have a scattering mean free path substantially longer than the shock thickness, then the simple test-particle picture of acceleration at a discontinuous velocity jump applies. These conditions are probably satisfied for ions which achieve a Lorentz factor of a few times that of the shock front. However, electrons are likely to have a more difficult task, since they must achieve a gyro radius in excess of the shock thickness, which is presumably governed by the ion gyro radius. In this respect it is interesting to note that the particle-in-cell simulations of Hoshino et al (1992) describe a mechanism which provides pre-acceleration of positrons in precisely this energy range.

Having arrived at a robust prediction, it is, of course, tempting to search for observational confirmation. The least controversial example of of a relativistic shock front is that thought to terminate the pulsar wind in the Crab Nebula (Rees & Gunn 1974). The electrons accelerated at this shock front are presumably responsible for the nebular radiation, which extends from the radio to the VHE gamma-ray range. According to Kennel & Coroniti (1984), the hard X-ray emission at 1–100 keV is emitted close to the shock front by cooling accelerated electrons. The spectrum required to fit the observations has a photon index close to 2.1 – precisely that expected if electrons are injected with the spectrum predicted for an ultra-relativistic shock front: $s = 4.2$. Different values of s seem to be indicated by observations of other X-ray nebulae around pulsars, but the models of these sources are much less well developed (Chevalier 2000). Ultra-relativistic shocks are also thought to manifest themselves in the afterglows of gamma-ray bursts. The observational situation here is less clear-cut: in GRB 970508, Galama et al (1998) deduce a photon index of 2.1, but analysis of the light curves favours a somewhat softer spectrum $s \sim 2.6$ (Sari, Piran & Narayan 1998).

In summary, there is a consensus concerning the predicted spectrum of particles accelerated at a relativistic shock front, but more work on modelling and more observations are needed before the theory can be tested.

References

Achterberg, A., Gallant, Y.A., Kirk J.G., Guthmann, A.W. 2001 submitted to MNRAS

Baring, M.G. 1999, 'Acceleration at relativistic shocks in gamma-ray bursts', Proceedings of the 26th ICRC, Salt Lake City 4, 5–8

Bednarz, J., Ostrowski, M. 1998, 'Energy spectra of cosmic rays accelerated at ultrarelativistic shock waves', Phys.Rev.Lett, 80, 3911–3914

Begelman, M.C., Kirk, J.G. 1990, 'Shock-drift particle acceleration in superluminal shocks: a model for hot spots in extragalactic radio sources', ApJ, 353, 66–80

Chevalier, R.A 2000, 'A Model for the X-Ray Luminosity of Pulsar Nebulae', ApJ, 539, L45–L48

Galama, T., Wijers, R.A.M.J., Bremer, M., Groot, P.J., Strom, R.G., Kouveliotou, C., van Paradijs, J. 1998, 'The Radio-to-X-Ray Spectrum of GRB 970508 on 1997 May 21.0 UT', ApJ, 500, L97–L100

Gallant, Y.A., Achterberg, A., Kirk, J.G. 1999, 'Particle acceleration at ultrarelativistic shocks – Gamma-ray burst afterglow spectra and UHECRs', A&AS, 138, 549–550

Gallant, Y.A., Achterberg, A., Kirk, J.G., Guthmann, A.W. 2000, 'Particle Acceleration at Ultra-Relativistic Shocks and the Spectra of Relativistic Fireballs', in ASP Conf. Ser. Vol. 526, Proceedings of the 5th Huntsville Gamma-Ray Burst Symposium, Ed: R.M. Kippen, R.S. Mallozzi & G.J. Fishman (New York, ASP) 524–529

Heavens, A.F., Drury, L.O'C. 1988, 'Relativistic shocks and particle acceleration', MNRAS, 235, 997–1009

Hoshino, M., Arons, J., Gallant, Y.A., Langdon, A.B. 1992, 'Relativistic magnetosonic shock waves in synchrotron sources — Shock structure and nonthermal acceleration of positrons', ApJ, 390, 454–479

Kennel, C.F., Coroniti, F.V. 1984, 'Magnetohydrodynamic model of Crab Nebula radiation', ApJ, 283, 710–730

Kirk, J.G., Duffy, P. 1999, 'Particle acceleration and relativistic shocks', J. Phys. G: Nucl. Part. Phys. 25, R163–R194

Kirk, J.G., Guthmann, A.W., Gallant Y.A., Achterberg, A. 2000, 'Particle acceleration at ultrarelativistic shocks: an eigenfunction method', ApJ, 542, 235–242

Kirk, J.G., Melrose, D.B., Priest, E.R. 1994, 'Plasma Astrophysics', Springer-Verlag, Berlin

Kirk, J.G., Schneider, P. 1987, 'On the acceleration of charged particles at relativistic shock fronts', ApJ, 315, 425–433

Michalek, G., Ostrowski, M. 1998, 'On the cosmic ray cross field diffusion in the presence of oblique MHD waves', A&A, 337, 558–564

Ostrowski, M., Bednarz, J. 2000, 'Comment on the first-order Fermi acceleration at ultra-relativistic shocks', to appear in Proceedings of the Heidelberg Gamma-Ray Symposium 2000

Rees, M.J., Gunn, J.E. 1974, 'The origin of the magnetic field and relativistic particles in the Crab Nebula', MNRAS, 167, 1–12

Sari, R., Piran, T., Narayan, R. 1998, 'Spectra and Light Curves of Gamma-Ray Burst Afterglows', ApJ, 497, L17–L20

Smolsky, M.V., Usov, V.V. 2000, 'Nonthermal Radiation of Cosmological Gamma-Ray Bursters', ApJ, 531, 764–775