

Logical systems and natural logical intuitions

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Abstract

The present paper is part of a large research programme investigating the nature and properties of the predicate logic inherent in natural language. The general hypothesis is that natural speakers start off with a basic-natural logic, based on natural cognitive functions, including the basic-natural way of dealing with plural objects. As culture spreads, functional pressure leads to greater generalization and mathematical correctness, yielding ever more refined systems until the apogee of standard modern predicate logic. Four systems of predicate calculus are considered: Basic-Natural Predicate Calculus (BNPC), Aritsotelian-Abelardian Predicate Calculus (AAPC), Aritsotelian-Boethian Predicate Calculus (ABPC), also known as the classic Square of Opposition, and Standard Modern Predicate Calculus (SMPC). (ABPC is logically faulty owing to its Undue Existential Import (UEI), but that fault is repaired by the addition of a presuppositional component to the logic.) All four systems are checked against seven natural logical intuitions. It appears that BNPC scores best (five out of seven), followed by ABPC (three out of seven). AAPC and SMPC finish *ex aequo* with two out of seven.

1. The programme

This tentative and exploratory paper is about a topic that has not been broached in the literature so far: the empirical adequacy of a logical system of predicate calculus. By this is meant the question of how well a logical system corresponds with natural speakers' intuitions about logical relations between and truth conditions for (propositions in) sentences. This is the methodological rationale of the research programme:

- Progress in science is contingent upon taking the data seriously.
- In the cognitive sciences, the data is experiential intuitions. In the semantics of natural language, data consists of logico-semantic experiential intuitions.
- It has been known for the last one hundred years, but one may also say for the last two thousand years, that accepted logical systems violate natural intuitions.
- This fact has not been taken seriously by logicians, whose perspective has always been metaphysical and/or mathematical, and not cognitively empirical.
- Pragmatics has served as palliative therapy.
- To understand the semantics of natural language it is necessary to investigate the possibility of reconstructing NATURAL HUMAN LOGIC.

2. What is logic? What is natural logic?

Logic is the formal study of consistency within a text (where "consistency" is taken in the sense of possible simultaneous truth). Therefore, logic is essential in the study of semantics: when we convey information, tell a story, issue an order or ask a question, we need to be consistent. Logic is *formal* by definition—that is, any logic is a calculus which, when followed, guarantees consistency for the sentences covered by it. For most forms of consistency, however, no calculus is available. Thus, we know that (1a) is inconsistent with (1b), but there is no logical calculus that proves it:¹

- (1)a. John speaks French.
- b. John died two years ago.

By contrast, (2a) is also inconsistent with (2b), but now we have a logic to prove it, as predicate logic tells us that (2b) entails the existence of at least one speaker of French, while (2a) blocks any such entailment:

- (2) a. Nobody speaks French.
- b. Some speakers of French live in London.

For natural intuition, the following two are also inconsistent:

- (3) a. Nobody speaks French.
- b. All speakers of French live in London.

But in standard modern predicate calculus (SMPC) (3a) and (3b) are taken to be consistent, because, in this logic, (3b) counts as true when there are no speakers of French.

The general hypothesis is that natural speakers start off with a basic-natural logic, based on natural cognitive functions, including the basic-natural way of dealing with plural objects. As culture spreads, stricter thinking, resulting from functional pressure, leads to better generalization and greater mathematical correctness, yielding ever more refined systems until the apogee of standard modern predicate logic. Individuals and societies are thus taken to be able to 'bootstrap' themselves up to higher levels of intellectual achievement.²

The Gricean maxims are meant to prevent this and similar clashes between intuitions and "official" logic, but their explanatory power turns out to be insufficient. Consider, for example, the following two sentences:

- (4)a. John could't go forward or backward.
- b. John could't go forward and backward.

1 One might think of a formal machinery computing entailments from lexical meanings—an implementation of the programme of "meaning postulates" as proposed in Carnap (1956). But no such machinery has as yet been made available. Even then, however, as argued in Seuren (in press, section 2.3.1), there still is the question of the overarching general logic to which the machinery of meaning postulates is subservient.

2 See Seuren 2006, in press Chapter 3. Similar processes have been observed for numeracy, reading and geometrical competence (Dehaene 1997, Dehaene 2005; Dehaene et al. 2006; Pica et al. 2004).

Sentence (4a) is immediately understood as ‘John couldn’t go forward *and* he couldn’t go backward’. Yet (4b), with *and* instead of *or*, is not immediately understood as ‘John couldn’t go forward *or* he couldn’t go backward’: one needs a considerable amount of sophisticated thinking to convince oneself that (4b) is equivalent with ‘John couldn’t go forward *or* he couldn’t go backward’. This is remarkable, because, on the assumption that natural logic equals standard logic, standard propositional logic predicts that the same processing procedure should hold for the two, both being instances of De Morgan’s laws of conversion between conjunction and disjunction. The Gricean maxims, however, have no bearing at all on such cases.

Given the (now more and more acknowledged) inability of the Gricean maxims to bridge the gap between standard logic and natural intuitions, we try a different approach: we say that natural language has its own logic, which differs from standard logic. To see how this can work, we must first see what defines a logic.

3. What defines a logic?

Aristotle (384–322 BCE) discovered that a formal theory of textual consistency crucially depends on a handful of words, called OPERATORS or CONSTANTS. For him—as for us—these words are ALL, SOME, NOT, AND, OR, IF (also MAY, MUST, and a few more, but we leave these out of account here). Logical formulae thus consist of operators and variables, the latter ranging over propositions (propositional logic) or over predicates (predicate logic). In any logic, the operators are defined as regards the conditions under which they produce truth (their satisfaction conditions). Standard logic uses satisfaction conditions that mirror standard set-theoretic operations.

The empirical question now is: how are the operators (or rather, the words that correspond to them) defined in natural language—assuming that individual languages do not differ in this respect? If we can define the natural-language logical operators in an empirically valid way, and if these definitions form a sound system of logic, we have the natural logic of language and cognition, which is likely to differ in important ways from standard logic. Logic thus becomes a matter of lexical semantics.

4 Four predicate logics

Predicate logic is the theory of (universal and existential) quantification. Given the three quantifiers ALL, SOME and NO, and the negation NOT, either over the whole sentence/proposition (external negation) or over the predicate (internal negation), we distinguish the following twelve basic sentence types (without vacuous repetitions of negations; the variables “F” and “G” stand for predicates):

| | | | |
|-----------|-----------------|------------|-----------------------|
| A | ALL F is G | ¬A | NOT [ALL F is G] |
| I | SOME F is G | ¬I | NOT [SOME F is G] |
| N | NO F is G | ¬N | NOT [NO F is G] |
| A* | ALL F is NOT-G | ¬A* | NOT [ALL F is NOT-G] |
| I* | SOME F is NOT-G | ¬I* | NOT [SOME F is NOT-G] |
| N* | NO F is NOT-G | ¬N* | NOT [NO F is not-G] |

In those logics where NOT-SOME = NO, the number is reduced to the following eight:

| | | | |
|-----------|-----------------|------------|----------------------------|
| A | ALL F is G | ¬A | NOT [ALL F is G] |
| I | SOME F is G | ¬I | NO (= NOT-SOME) F is G |
| A* | ALL F is not-G | ¬A* | NOT [ALL F is NOT-G] |
| I* | SOME F is NOT-G | ¬I* | NO (= NOT-SOME) F is NOT-G |

This ‘language’ will do for the present purpose. But what do the words ALL, SOME, NO and NOT mean? Simplifyingly, we take NOT to be the standard truth-value toggle in all four logics considered. As regards ALL, SOME and NO, they are defined in SMPC as follows (“ $\llbracket P \rrbracket$ ” stands for the *extension* of the predicate P; “ $\langle a, b \rangle$ ” stands for the ordered pair ‘a followed by b’):

(5) **Standard Modern Predicate Calculus SMPC:**

$$\llbracket \text{ALL} \rrbracket = \{ \langle \llbracket F \rrbracket, \llbracket G \rrbracket \rangle \mid \llbracket F \rrbracket \subseteq \llbracket G \rrbracket \}$$

(the extension of the predicate ALL consists of the set of all pairs $\llbracket F \rrbracket$ and $\llbracket G \rrbracket$ such that $\llbracket F \rrbracket$ is included in or equals $\llbracket G \rrbracket$)

$$\llbracket \text{SOME} \rrbracket = \{ \langle \llbracket F \rrbracket, \llbracket G \rrbracket \rangle \mid \llbracket F \rrbracket \cap \llbracket G \rrbracket \neq \emptyset \}$$

(the extension of the predicate SOME consists of the set of all pairs $\llbracket F \rrbracket$ and $\llbracket G \rrbracket$ such that the intersection of $\llbracket F \rrbracket$ and $\llbracket G \rrbracket$ is nonnull)

$$\llbracket \text{NO} \rrbracket = \{ \langle \llbracket F \rrbracket, \llbracket G \rrbracket \rangle \mid \llbracket F \rrbracket \cap \llbracket G \rrbracket = \emptyset \}$$

(the extension of the predicate NO consists of the set of all pairs $\llbracket F \rrbracket$ and $\llbracket G \rrbracket$ such that the intersection of $\llbracket F \rrbracket$ and $\llbracket G \rrbracket$ is null)

(One notes that, under these definitions, NO F is G is simply the negation of SOME F is G.)

The relation with standard set theory is obvious. SOME simply requires nonnullness of the intersection of $\llbracket F \rrbracket$ and $\llbracket G \rrbracket$, whereas ALL requires inclusion (in the standard sense) of $\llbracket F \rrbracket$ in $\llbracket G \rrbracket$. For the rest, truth is determined by the laws and theorems of standard set theory. Thus, in SMPC, ALL F is G is trivially true when $\llbracket F \rrbracket$ is null ($\llbracket F \rrbracket = \emptyset$), because in standard set theory the null set (\emptyset) is a subset of any set. And SOME F is G is trivially true when both $\llbracket F \rrbracket$ and $\llbracket G \rrbracket$ are nonnull and $\llbracket F \rrbracket \supseteq \llbracket G \rrbracket \neq \emptyset$, because when these conditions are met, their intersection is nonnull.

It is well-known, however, that natural intuition does not support such truth judgements. For example, ALL F is G is false for natural intuition when there are no Fs and SOME F is G is likewise considered false when either $\llbracket F \rrbracket \subseteq \llbracket G \rrbracket$ (so that ALL F is G is true), but true when $\llbracket G \rrbracket \subset \llbracket F \rrbracket$. Aristotle, followed by Abelard (1079–1142), respected the former intuition but not the latter. Their logic, AAPC, is a perfectly sound alternative to SMPC, from which it differs only in that, in the absence of any Fs, ALL F is G is considered false in the former but true in the latter. In AAPC, ALL is characterised by the following definition, while SOME and NO are as in (5):

(6) **Aristotelian-Abelardian predicate calculus (AAPC):**

$$\llbracket \text{ALL} \rrbracket = \{ \langle \llbracket F \rrbracket, \llbracket G \rrbracket \rangle \mid \llbracket F \rrbracket \neq \emptyset \text{ and } \llbracket F \rrbracket \subseteq \llbracket G \rrbracket \}$$

(the extension of the predicate ALL consists of the set of all pairs $\llbracket F \rrbracket$ and $\llbracket G \rrbracket$ such that $\llbracket F \rrbracket$ is nonnull and $\llbracket F \rrbracket$ is included in or equals $\llbracket G \rrbracket$)

$$\llbracket \text{SOME} \rrbracket = \{ \langle \llbracket F \rrbracket, \llbracket G \rrbracket \rangle \mid \llbracket F \rrbracket \cap \llbracket G \rrbracket \neq \emptyset \}$$

(the extension of the predicate SOME consists of the set of all pairs $\llbracket F \rrbracket$ and $\llbracket G \rrbracket$ such that the intersection of $\llbracket F \rrbracket$ and $\llbracket G \rrbracket$ is nonnull)

$$\llbracket \text{NO} \rrbracket = \{ \langle \llbracket F \rrbracket, \llbracket G \rrbracket \rangle \mid \llbracket F \rrbracket \cap \llbracket G \rrbracket = \emptyset \}$$

(the extension of the predicate NO consists of the set of all pairs $\llbracket F \rrbracket$ and $\llbracket G \rrbracket$ such that the intersection of $\llbracket F \rrbracket$ and $\llbracket G \rrbracket$ is null)

(Again, NO F is G is simply the negation of SOME F is G.)

This formulation differs from (5) only in that there is the extra requirement for the truth of **A** (ALL F is G) that $\llbracket F \rrbracket$ be nonnull. The logic resulting from (6) we call Aristotelian-Abelardian predicate calculus or AAPC.

But this fails to do justice to the other intuition that **I** (SOME F is G) is considered false when either $\llbracket F \rrbracket \subset \llbracket G \rrbracket$ or $\llbracket F \rrbracket = \llbracket G \rrbracket$, but true when $\llbracket G \rrbracket \subset \llbracket F \rrbracket$ ($\llbracket G \rrbracket$ being nonnull). Our intuition tells us that SOME is to be read as ‘some but not all’. To account for both intuitions, we define ALL, SOME and NO as follows:

(7) **Basic-Natural Predicate Calculus (BNPC):**

$$\llbracket \text{ALL} \rrbracket = \{ \langle \llbracket F \rrbracket, \llbracket G \rrbracket \rangle \mid \llbracket F \rrbracket \neq \emptyset \text{ and } \llbracket F \rrbracket \subset \llbracket G \rrbracket \}$$

(the extension of the predicate ALL consists of the set of all pairs $\llbracket F \rrbracket$ and $\llbracket G \rrbracket$ such that $\llbracket F \rrbracket$ is nonnull and $\llbracket F \rrbracket$ is included in or equals $\llbracket G \rrbracket$)

$$\llbracket \text{SOME} \rrbracket = \{ \langle \llbracket F \rrbracket, \llbracket G \rrbracket \rangle \mid \llbracket F \rrbracket \cap \llbracket G \rrbracket \neq \emptyset \text{ and } \llbracket F \rrbracket \cap \llbracket G \rrbracket \subset \llbracket F \rrbracket \}$$

(the extension of the predicate SOME consists of the set of all pairs $\llbracket F \rrbracket$ and $\llbracket G \rrbracket$ such that the intersection of $\llbracket F \rrbracket$ and $\llbracket G \rrbracket$ is nonnull and is properly included in $\llbracket F \rrbracket$)

$$\llbracket \text{NO} \rrbracket = \{ \langle \llbracket F \rrbracket, \llbracket G \rrbracket \rangle \mid \llbracket F \rrbracket \cap \llbracket G \rrbracket = \emptyset \}$$

(the extension of the predicate NO consists of the set of all pairs $\llbracket F \rrbracket$ and $\llbracket G \rrbracket$ such that the intersection of $\llbracket F \rrbracket$ and $\llbracket G \rrbracket$ is null)(Now, NO F is G is not the negation of SOME F is G because NO F is G and SOME F is G are both false in cases where ALL F is G is true.)

This formulation differs from (6) only in that ALL requires *proper* inclusion of $\llbracket F \rrbracket$ in $\llbracket G \rrbracket$ and that **I** (SOME F is G) requires for truth not only that the intersection of $\llbracket F \rrbracket$ and $\llbracket G \rrbracket$ be nonnull but also that this intersection be properly included in $\llbracket F \rrbracket$. The logic resulting from (7) we call Basic-Natural Predicate Calculus or BNPC.³

There is a fourth predicate logic, the famous SQUARE OF OPPOSITION, often falsely attributed to Aristotle (whose system of predicate logic is AAPC as defined above). The SQUARE is the product of Aristotle’s commentators Apuleius (±125–180), Ammonius (±440–520) and Boethius (±480–524), who meant to streamline Aristotle’s original system AAPC, thereby unwittingly introducing the logical error of *undue existential import* (UEI). In the Square, the definitions of ALL, SOME and NO are as in

3 BNPC is essentially the same as the predicate logic developed by the Edinburgh philosopher Sir William Hamilton (1788–1856), if one forgets about Hamilton’s insistence on ‘quantification of the predicate’, which is not discussed in the present paper (see Seuren, in press, section 3.4.2).

(6), but the commentators added the theorem of the so-called Conversions, which holds in SMPC but not in AAPC and even less in BNPC. The Conversions are defined as follows:

$$(8) \quad \mathbf{A} \equiv \neg \mathbf{I}^* \quad \text{and} \quad \mathbf{I} \equiv \neg \mathbf{A}^*$$

That is, ALL and SOME are interchangeable provided an external and an internal negation are added. In current terminology it is said that ALL and SOME are *duals* in the systems concerned.

The Conversions do not follow from the semantic definitions of the quantifiers given in (6) but were added as independent elements, which is why they have led to the logical error of UEI: a sound logic is defined exclusively by the semantics of its operators (constants); any further additions may make a logic unsound. In the case of the Square, the logical defect of UEI is not too serious, as it can be eliminated by the addition of a *presuppositional component* to the logic, which then contains the Square as a proper subpart (see Seuren 1988; Seuren, in press Chapter 10).

Apart from the Square, which suffers from the logical defect of UEI, the other three systems are logically sound. The soundness of SMPC cannot be at issue. AAPC is as sound as SMPC.⁴ BNPC as defined in (7) suffers from UEI only when situations where $\llbracket \mathbf{F} \rrbracket = \emptyset$ are left out of account (as they should be in a fully natural logic). But when situations where $\llbracket \mathbf{F} \rrbracket = \emptyset$ are taken into account, BNPC is again sound (see Seuren, in press, Chapter 3). BNPC suffers from the functional but nonlogical disadvantage that **I**-type sentences are like **A**-type and **N**-type sentences in that their truth cannot be established until the whole universe of objects **U** has been checked—a limitation that is crippling when **U** is infinite. In the case of **A**-type and **N**-type sentences, at least their falsity is established when a counterexample is found, but **I**-type sentences are in principle unascertainable in an infinite **U**. This disadvantage, however, is not of a logical but only of a practical nature.

5 Intuitions tested

These four systems are now set off against the following seven natural logical intuitions (“ \vdash ” or “entails”, here: “is felt to entail”; likewise for “ \equiv ” or “is equivalent”, here: “is felt to be equivalent”):

- | | |
|---|---|
| 1. SOME F is G \vdash NOT [ALL F is G] | $\mathbf{I} \vdash \neg \mathbf{A}$ |
| 2. SOME F is G \equiv SOME F is NOT-G | $\mathbf{I} \equiv \mathbf{I}^*$ |
| 3. SOME F is G \equiv SOME G is F | $\mathbf{I} \equiv \mathbf{I}!$ |
| 4. ALL F is G \vdash SOME G is F | $\mathbf{A} \vdash \mathbf{I}!$ |
| 5. ALL F is G \vdash SOME G is NOT-F | $\mathbf{A} \vdash \mathbf{I}!^*$ |
| 6. NO F IS NOT-G \equiv ALL F IS G | $\mathbf{N}^* \equiv \mathbf{A}$ |
| 7. NOT-ALL F is G \equiv SOME F is G \equiv SOME F IS NOT-G | $\neg \mathbf{A} \equiv \mathbf{I} \equiv \mathbf{I}^*$ |

The combination of 3 and 4 amounts to the *positive subaltern entailment* from ALL F is G to SOME F is G, found in AAPC and the ABPC (the Square) but not in BNPC or

4 The soundness of, in particular, AAPC may give standard logicians pause to think (see Seuren, to appear).

SMPC. (The positive subaltern entailment schema is not fully or basically natural but is defensible on further reflection.) The four systems score as follows:

| | |
|------|------------------------------|
| BNPC | 1, 2, 4, 5, (6) ⁵ |
| AAPC | 3, 4, |
| ABPC | 3, 4, 6 |
| SMPC | 3, 6 |

BNPC scores best: it misses out only on the intuitions 3 and 7. AAPC and the ABPC successfully account for the intuitions 3 and 4. In both systems SOME F is G is symmetrical (SOME F is G \equiv SOME G is F) and ALL F is G \vdash SOME G is F, because when $\llbracket F \rrbracket \subseteq \llbracket G \rrbracket$ and $\llbracket F \rrbracket \neq \emptyset$, $\llbracket F \rrbracket \cap \llbracket G \rrbracket \neq \emptyset$. ABPC has the extra advantage of accounting for intuition 6, since in that system NO F IS NOT-G and ALL F IS G are equivalent in virtue of the Conversions. Standard modern predicate calculus (SMPC) accounts for the intuitions 3 (SOME is symmetrical) and 6 (NO F IS NOT-G \equiv ALL F IS G in virtue of the Conversions).

None of the systems is able to account for intuition 7. ABPC and SMPC come closest in that there NOT [ALL F is G] \equiv SOME F is NOT-G, but SOME F is G and SOME F is NOT-G are not equivalent in these systems. In BNPC, when NOT [ALL F is G] is true, then either SOME F is G (=SOME F is NOT-G) or NO F is G is true, but the latter possibility is so counterintuitive as to be semantically abhorrent. In AAPC, SOME F is NOT-G \vdash NOT [ALL F is G] but not vice versa, because when $\llbracket F \rrbracket = \emptyset$, NOT [ALL F is G] is true but SOME F is NOT-G is false.

The explanation proposed falls back on topic-comment structure (information structure). We formalise topic-comment structure as an underlying cleft structure where *all* has comment status. It is assumed that the default analysis of a sentence like *Ben didn't eat all of his meal*, is 'what Ben ate of his meal was not all', entailing presuppositionally that Ben ate some of his meal and thus excluding the case that he ate nothing. Thus read, the sentence PRESUPPOSES that Ben ate some of his meal and ASSERTS that he did not eat all of it. Similarly for (9a), which, if analysed as (9b), excludes the case that there are no green flags:

- (9)a. Not all flags are green.
- b. 'the flags that are green are not all (flags)'

Thus, to the extent that NOT-ALL denies the comment ALL, the intuitive equivalence of SOME F is (NOT-)G with NOT-ALL F is G is explained by topic-comment structure.

It seems that we have to conclude that the totality of natural logical intuitions held by logically naive humans does not fit into a single logical system. In order to account for all the intuitions, a distinction will have to be made between a *basic-natural* and a *strict-natural* system of predicate logic.

5 The equivalence expressed in Intuition 6 follows from BNPC only when situations where $\llbracket F \rrbracket = \emptyset$ are left out of account, as they are by those who operate with BNPC as 'their' predicate logic.

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